Continuity Equation

SPEAKER Bob Holmes: Okay. Now we've put aside the concept of streamlines, and we're gonna talk about the continuity equation. Now the principle of conservation of mass is that matter can neither be created nor destroyed. We're gonna use that principle and apply that to come up with the continuity equation or the equation of continuity on a stream tube.

On your screen, you're seeing a free-body diagram where I've got a--particular looking from the top, so it's in plain view and I'm looking down on a section of stream. And this is a well-behaved stream. It's got nice linear sides and it expands out. So you've got a smaller section that expands into a larger section. If I use a control volume, remember we talked about free-body diagrams, we'll also use that kinda interchangeably with something in fluid mechanics we call a control volume. And if I look at the control volume as being from Section B through the expansion to Section B-prime, that is my control volume at the beginning of my simulation here. So I've got a control volume in section--at Point B. It has a velocity of V1. In the expansion, along Section B-prime or at the location of B-prime, I have a velocity of V2. I wanna note that all points in this pipe, the particles of the fluid move tangential to the streamline, which we would expect to happen in steady flow, if you paid attention to the previous section where I defined that the streamlines would be all tangential to the--or the velocity would all be tangential to the streamlines for steady flow. Now, as I move through this process in a small interval of time, delta T, I have fluid that begins at the--as I look down in cross-section at B to B-prime, that fluid and that control volume, that free body, I'm going to move it ahead in space, and it's now going to occupy the location of C to C-prime in my diagram.

Now, if I look and I say, "Okay, how far is that fluid going to move from B to C?" I can say that that distance, delta S1, is equal to the velocity V1 times my differential time, my small increment of time, delta T. The cross-sectional area of that particular location is A1. Now, if I know--wanna know exactly how much mass is moved, and you'll see that being illustrated on your diagram, I'm going to move that mass B-C--the mass is defined by B to section C. That mass is moving out of that and it's going to move further down through my cross-section into the next area, that's gonna

be the mass that's going to be in B, C-prime, B-prime, C-prime.

Now, if I define the mass in that Section 1 as being the density. Remember the density is a Greek letter rho, and that has units of slugs per volume. If I multiply that times the volume of that particular area, which will be delta S times A1, that would give me the mass that moved out of that particular location. Because of conservation of mass, I know that the amount of mass that moved in to section B-prime, C-prime has to equal what moved out of Section B-C.

Now, the mass in Section 2, that B-prime, C-prime area, section, that's going to be equal to the density, again, the Greek letter rho, times the cross-sectional area A2 times delta S2. Now you'll note that delta S1 and delta S2 are different. And why is that so? It's because the velocities are different. You can't think of that conceptually. You see, if you constrict the channel, you know that the velocity moves a lot faster there than it does where it takes--then is contrasted where to expand it. And so you would assume that a particle moving at a slower velocity, if we look at it just from a particle sense, it's gonna move a lot farther in a particular time in a faster section than it would in the expanded section. So, you can see that.

Now, if I equate the two masses, mass in Section 1 equal to the mass in Section 2, I set that rho 1, A1, B1 times delta T is equal to rho 2, A2, B2 times delta T, I come up with--through some algebraic manipulation, I get rid of the densities on each side and I get rid of the delta T's, I come up with the equation A1 times V1 is equal to A2 times V2. We know that the cross-sectional area times the velocity is equal to volumetric discharge Q. And if you've ever made a discharge measurement, you've applied this concept already because you've gone through and you've measured the cross-sectional area, you've measured the velocities, you multiplied those by each other and you came up with a volumetric discharge in units of volume per unit time or cubic feet per second in most of our applications.

That concludes are discussion on the continuity equation. We're gonna use that along with the energy equation to solve a number of problems throughout this particular course.