Derivation of Type 1 Flow Equation

SPEAKER Rick Huizinga: In this section, I'd like to derive for you the discharge equation for the type 1 culvert flow. And on the screen in front of you, you have a diagram that shows you the definition of a type 1 culvert. Once again, critical depth occurs at the inlet; h sub-1, which is the distance of--the elevation of the upstream water surface in relation to the downstream, the tailwater invert, is greater than the critical depth, and the depth in the culvert is less than the critical depth, so we have super-critical flow in the culvert. The slope of the culvert is at a steep slope.

As shown in the earlier section, transition from sub to supercritical is a good point of flow control and it's also a good point to start computations for discharge. So let's write the energy equation from section 1 to section 2. We have h1 plus alpha1 V1 squared over 2g is equal to d2 plus z plus V2 squared over 2g plus the entrance losses plus the friction losses between sections 1 and 2. Now we assume alpha2 to be 1 because it is generally a constant cross-section and we don't have to worry about Coriolis coefficients there.

Now the entrance loss h sub-e is taken as k sub-e times V2 squared over 2g. And that k sub-e term can range from 0.1 to 0.6 depending on the entrance shape. If it's rounded or bell-shaped, it is more efficient; if it's square and boxy, it's less efficient. So let's rewrite the energy equation in terms of V2 squared. We can take the first portion and we see V2 squared over 2g plus k sub-e V squared over 2g is equal to h1 minus z plus alpha1 V1 squared over 2g minus d2 minus the friction loss from section 1 to 2. But we know that V2 is equal to Q over A, and since critical depth occurs at the inlet, then A2 is equal to A-critical and D2 is equal to D-critical. So Q squared over A squared 2g times the quantity 1 plus k sub-e is equal to h1 minus z plus alpha1 V1 squared over 2g minus d2 minus h sub-f from 1 to 2. Now let's define a coefficient of discharge, C, such that C is equal to 1 over the square root of 1 plus k sub-e. If we put that back into our equation then, we see that we get Q squared over A squared times 2g, times the quantity 1 over C squared is equal to h1 plus z--excuse me--h1 minus z plus alpha1 V1 squared over 2g minus d2 minus h sub-f from 1 to 2, the friction loss from 1 to 2.

We solved all of that for Q; we then see that Q squared is equal to A-c squared times C squared times 2g times the quantity h1 minus z plus alpha1 V1 squared over 2g minus D2 minus h sub-f

from 1 to 2. Or taking the square root of all of those things, Q is equal to A sub-c times C times the square root of the quantity 2g times h1 minus z plus alpha1 V1 squared over 2g minus D2 minus h sub-f from 1 to 2. Now, this matches the equation that we've already shown you for type 1 culvert flow. The coefficient of discharge C is determined from lab experimentation and more extensive tables and charts for C are found in Bodhaine which is the reference for the TWRI that I've already referenced in the earlier section and you can find in the resource section of this course.

Now, let's talk a little bit about the head loss in the type 1 equation. The head loss between sections 1 and two h sub-f from 1 to 2 is equal to the length, L, times the Q squared over conveyance at section 1 times conveyance at section 2 where L is the distance from 1 to 2. Well, where does this come from? Well, remember from Lessons 13 and 15 that when we use the geometric mean to determine head loss, h sub-f is equal to the average friction loss S sub-f bar times the length, and that the average friction slope S sub-f bar is equal to the square root of the friction loss at the downstream end times the friction loss at the upstream end.

Well, remember from Lesson 8, Q is equal to conveyance times the square root of the friction slope or the square root of the friction slope is equal to Q over K. So if we were to re-write h sub-f in terms of length times the friction slope from 1 to 2, the average friction slope from 1 to 2, we see that we can re-write that as L times the square root of S sub-f1 times S sub-f2 which is equal to L times the square root of S sub-f times the square root of S sub-f. So when we re-write all of that in terms of Q's and conveyances, the end result is that the friction loss from 1 to 2 is equal to the length times the Q squared over conveyance1 times conveyance2.

There are a couple of other terms that we need to be able to solve this equation. The energy equation from 1 to 2 once again is h1 plus alpha1 V1 squared over 2g and that's equal to D2 plus z plus V2 squared over 2g plus k sub-e times V2 squared over 2g plus h sub-f from 1 to 2. If we rearrange that then, we move D2 and V2 over to the one side, we get D2 plus V2 squared over 2g is equal to h1 plus alpha1 V1 squared over 2g minus z minus k sub-e times V2 squared over 2g minus the friction loss from 1 to 2. Note that the first term is the specific energy at section 2 or

Derivation of Type 1 Flow Equation

E2 is equal to h1 plus alpha1 V1 squared over 2g minus z minus k sub-e V2 squared over 2g minus h sub-f from 1 to 2.

Now let's define a new term, H1, and we'll call that the specific energy at section 1 relative to the bottom of the culvert entrance. So H sub-1 is equal to h sub-1 plus alpha1 squared V1 h--H1 is equal to h1 plus alpha1 V1 squared over 2g minus z. Now if we use that in our previous equation, we see that E2 is equal to H1 minus k sub-e V2 squared over 2g minus the friction loss, h sub-f, from 1 to 2. Now we need the critical depth in the culvert, D sub-c, to solve the equation and we know a few things about critical depth. First of all, we know the Froude number is equal to one at critical depth and the Froude equation V sub-c over the square root of gD sub-c. So we can re-write that and we know then that D sub-c over 2 is equal to the critical velocity head, V sub-c, squared over 2g. And recall from Lesson 11 that D sub-c is also equal to two-thirds of the critical energy, E sub-c. So in other words, E sub-c is equal to three halves times D sub-c. So if we put these into the previous equation, we can replace E2 with three halves D sub-c and we can replace the V squared over 2g with D sub-c over 2. So you get the equation on the lower right-hand side of your screen. Three halves D sub-c is equal to H1 minus k sub-e times D sub-c over 2 minus the friction loss h sub-f from 1 to 2. If we solve that for D sub-c, we see that we have H sub-1 minus the friction loss, h sub-f, from 1 to 2 is equal to three halves D sub-c plus k sub-e times D sub-c over 2. Or simplifying, you get the equation in the middle of the slide, D sub-c is equal to H1 minus the friction loss, h sub-f, from 1 to 2 all over the quantity 1.5 plus 0.5 times k sub-e. Now, this is the true critical depth of the inlet and it considers friction loss and entrance losses.

If we were to set the friction loss and the entrance losses to 0 such that h sub-f from 1 to 2 is 0 and k sub-e was equal to 0, you would see that D sub-c would be equal to two-thirds times H sub-1, which would imply that H sub-1 would be equal to the critical energy at the inlet to the pipe E sub-2. However, that can never be true because there are always friction and entrance losses. Therefore, there's a ratio of D sub-c to H1 and this is called the D sub-c factor and it's computed from the coefficient of discharge, C. You can find this D sub-c factor on page 25 of TWRI that references culvert flow.

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This is the end of the lecture portion of Lesson 17. And in the next section, we'll use the flume to demonstrate several of the different types of culvert flow.