Determination of Bed Shear Stress

SPEAKER Bob Holmes: We're going to talk about bed shear stress in this section and the various ways that you would compute it. As I've mentioned before, it's very crucial to understand the concepts of bed shear stress. Bed shear stress is the predominant mechanism that we have influencing the energy loss, it is essentially how the water interacts with the boundary. It's one of the crucial things to understand as part of this class.

Now, on your slide you're going to see a cross-section view of a channel. And as I've mentioned in the introduction, we're going to talk about wetted perimeter. And you can see that the wetted perimeter, which is abbreviated capital P, is that area or that length which is where the water surface is in contact with the boundary. So as you go from the water surface at the top where it interacts with the cross-section or the channel cross-section, you trace that in a linear fashion all the way around the channel. If you have a rectangular channel, that would be the depth on each side and the cross-sectional width. If you add those numbers together, that would give you the wetted perimeter.

Now, we're going to talk about a new term: prismatic. In a lot of our cases, especially early on in this course, we're talking about prismatic channels just to make the computations a little simpler. Whenever you hear somebody use the word prismatic, that means that the cross-section does not vary along the channel. For example, if you had a rectangular channel of about 10 to 15 feet wide and you had side walls that were, again, vertical because it's rectangular, that width would not change all the way through this prismatic section. So that's what we mean when we talk about prismatic channel.

Now, we also are going to introduce the concept of hydraulic radius, the hydraulic radius which is sometimes abbreviated just capital R, sometimes capital R sub-h, depends on the textbook that you use. In this course, we're going to use predominantly capital R. That is the cross-sectional area divided by the wetted perimeter. That is the definition of hydraulic radius. Again, that's cross-sectional area, capital A, divided by the wetted perimeter, which is abbreviated capital P.

Now, on your next slide, you're going to see a fairly busy cross-section or a side view of a prismatic

channel. And, essentially, what we have here is we have an X-axis that's going to align parallel to the bottom of the channel, and you'll see that abbreviated small letter "x," you have a Y-axis that is normal to the bed of the channel. Again, that's not vertical as in to gravity but we're actually aligning it normal to the channel bottom. So you see that slightly slanted in your slide. You have a cutaway, so this is a free-body diagram. Remember, when we talked about free-body diagrams early on in lesson one? And so where we cut on the upstream and downstream side, we have to replace those with the forces. We have the pressure force on the upstream side, abbreviated capital F sub-1. We have the downstream pressure force opposing the flow. That is abbreviated capital F sub-2. You'll notice the lines that are there, that's the pressure prism that we talked about in Lecture 2, and that essentially is--shows a linear increase of the pressure with the depth and so there you have the resultant force, F1 and F2, on your drawing.

We also have a weight force. That's the weight of the fluid in the channel, and that's acting vertically down. You'll note that we will have to resolve those components into X and Y components on your graph there. On the diagram, we'll put it in X and Y components so we have to know what that angle theta is. That angle theta is, just by the geometry of the problem, is also the angle theta that the channel bed is inclined with the horizontal.

Now, you also will note that we have some distance L that is along the horizontal. And in another graph here, in the second that I'll pop up, we also have a distance that will be along the channel itself. But we'll note that because the slope of the channel is usually very small, that L is approximately equal to that longitudinal distance along the channel itself. We have the energy grade line that's drawn where you can see that the kinetic energy head or the velocity head is above the water surface, V1 squared over two G on the upstream section and V2 squared over two G on the downstream section. We also have abbreviated in there the friction slope, S sub-f, and we'll show you what that looks like, and then lastly we have the head loss from Section 1 to 2 that's abbreviated small letter "h" sub-l and then in superscript we have 1-2.

Now, we're going to apply Newton's Second Law. And, again, Newton's Second Law says that the sum of the forces in each direction or each component is equal to the mass times the

acceleration. In this particular slide, we've abbreviated that in capital I where we're using the acceleration of that water mass to be abbreviated by the capital I. Now, as we again show you the diagram, you'll note that we have the force on the upstream end, which is the hydrostatic force due to the pressure, and we have the hydrostatic force on the downstream end and then along the bed we're going to have a friction force, a frictional resistance between the water and the boundary and then we'll have the weight component in the X-direction and the Y-direction due to the weight of the fluid itself.

Now, as we apply this Newton's Second Law, we can see that in the X-direction, which again is not horizontal, it is slightly inclined because we have a slope to this channel. So we're going to sum that in the X-direction, and we'll say that that's equal to the mass times the acceleration I equals to plus F1. So plus is to the right. Negative would be to the left, just like normal conventions in orthogonal coordinates. And so you're going to have F1 going to the right. F2 is a negative, so it's minus-F, the hydrostatic force on the downstream end, F2. You're going to add the weight component, but we're going to have to take the X-component, or you add the weight but you have to take the X-component of that weight. Essentially, that's going to be times the sine of theta, just do the geometry. And you might wanna consult a geometry textbook or ask somebody in your office that is--if you're not familiar with resolving components in X- and Y-directions using geometrical concepts, you'll need to review that. Now, because the slope of the bed is small, we could substitute instead of using the sine of theta, we could use the bed slope, S sub-zero.

And lastly, we're going to have the resistance force, and that's basically going to be the bed shear stress, which is the Greek letter tau sub-zero, multiply it by the area of the water that's in contact with the boundary. And we're going to use that value to be capital P, which is the wetted perimeter times the distance L. Now, on your graph in front of you, you have a capital K up there, which is showing you, as I've mentioned prior--in a prior side, that we didn't have it on that, but essentially that K is the linear distance along the channel. And so that's slightly inclined with the horizontal. However, because our slopes are usually very small, even if you have five to six

degrees, your cosine of that value is almost approximately equal to one. And so we can substitute instead of using K, we can just simply use the linear distance and the horizontal L. So we're going to use minus the wetted perimeter times the value L, which is the distance between the upstream and downstream components which defines our control volume, times that bed shear stress tau sub-zero.

Now, let's set this force balance equation aside for a moment and let's look at the shear forces on the boundary. The energy expended to overcome friction when the control volume moves a small distance, and we're going to abbreviate that D sub-s, small "d' sub--not sub-s, but DS. Delta S is another way to term that. It's the energy loss. It's also called head loss, and so we compute that as A sub-zero times tau sub-zero times that distance D sub-s--or DS. I apologize. I'm going to always have a little trouble with that. It's DS. Now, that A zero is the area that is in contact between the water surface and the boundary--or not the water surface, but the water and the boundary in that control volume. It's not to be confused with the cross-sectional area. That's the water in contact with the boundary. So it's the entire length of the control volume. So instead of having A sub-zero, we're going to write that as the wetted perimeter times L. Now, I've talked about that on the previous slide.

So the energy expended would be that shear force, wetted perimeter, P, times L times tau sub-zero, that's the shear force, times the distance that you would actually move it in a short period of time. So DS is how far we move it, and that would also correspond to the energy expended to move that control volume a distance, DS.

Now, we need to recall that in most of the stuff that we've talked about in Lecture 4, especially in Lecture 6, we've talked in the energy equation, we're always dealing with energy per pound to flowing fluid in the control volume. Therefore, the head loss occurring over a distance, DS, divided by the weight of the fluid in the control volume is that energy loss per pound to flowing fluid. Let's look at that in equation form. That would be the head loss, again abbreviated small letter "h" sub-l, is equal to the energy expended for that control volume to move a small distance, DS, and that would be the wetted perimeter times the length of that control volume, L, times the

value of the boundary shear stress, tau sub-zero, times the D sub-s--or DS, I'm sorry about that, and you're going to divide that by the weight of the control volume, which is Greek letter gamma, which again is the unit weight of water, times again that distance length of the control volume times the cross-sectional area A. Again, this is the cross-sectional area A, not A sub-zero, which we were talking about earlier, which was the area in contact with the bed--of the water in contact with the bed.

Now, we can simplify this, do some dividing, getting rid of the common terms. That becomes tau zero times DS divided by rho G times the cross-sectional area divided by the wetted perimeter. Again, what I've done here is I've expanded gamma, which is the unit weight of water. Another way to write that would be the density, rho, times G. So don't get confused there. I just put that in the bottom side of the denominator. After further manipulation, I can say that instead of using A over P, I can just abbreviate that with a hydraulic radius because, again, that's the definition of the hydraulic radius. So you can see that on the right side of your equation.

Now, to get to the slope of the energy grade line, because that's what we're going to be after, this S sub-f, whenever you're working these indirect computations of discharge or you're going to be looking at a step backwater model such as HEC-RAS in steady state mode, S sub-f is going to be the critical, crucial thing. So we're going to look at how you get to that point. And when we get later on in the next section, S sub-f is going to be a part of the equation for Manning equation. Okay? So we really need to look at it in more detail and be able to understand that. So we wanna look at that slope with the energy grade line. So we--in order to get that, we need to divide the head loss over that small distance, DS, by how far it occurs, which is again, DS. So if we say on our equation we have S sub-f, which is the friction slope, is equal to the head loss, H sub-l, divided by DS. Now, if we expand from a previous equation, we know what that head loss looks like. That's basically the bed shear stress tau zero times the distance DS divided by rho GR, which again that's the density of the fluid, times the acceleration of gravity times the hydraulic radius, and we divide that by that distance DS, which is still in the equation. Simplifying that further, we can see that that's simply tau zero divided by rho GR.

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Now, rearranging that, you can see the next equation that you have is the bed shear stress tau sub-zero is equal to rho GR S sub-f or, again, we know that rho G is simply just the unit weight of water, gamma, times R times S sub-f. Now, tau sub-zero is the bed shear stress, which you'll hear mentioned in some cases as the tractive force. Those are interchangeable terms.

Now, you'll also remember from Lecture 5 and when we derive the velocity profile equations, you'll notice that this is very similar to what was used there where we had it abbreviated or we had the equation as tau zero is equal to gamma times the depth of flow times sine of theta. And that was essentially if we have a rectangular channel or a unit width of water like we were using in the derivation of the velocity profiles, the hydraulic radius is approximately equal to the depth when you have a rectangular channel that's very wide. All right? So that's where that comes from. And we'll talk more about those kinds of assumptions, because whenever you deal with open channel flow problems, if you could have a large, wide channel instead of going through all the pain of calculating hydraulic radius, sometimes you can estimate that by just using the depth if it is approximately rectangular in its form. And many times on these large rivers, you can get away with that assumption.

Now, if we take the square of each side of that equation, we can say that the square root of tau zero over rho is equal to square root of GR S sub-f. Now, also from Lesson 5, you remember we introduced the concept of the shear velocity. This is where it comes from. And so now we have kind of come full circle here to where we're back again where we have the definition of the shear velocity being the tractive force or the bed shear stress, tau sub-zero divided by rho, that square root of that quantity is equal to the square root of GR times the friction slope S sub-f.

Now, if energy expenditures are due to other factors in addition to the boundary friction, you should be very careful. The relation between bed shear stress and friction slope is not valid. All right? So where would we put that? That's a long sentence there. It sounds like a lot of techno speak. What do I mean by that? Essentially, if I've got a nice boundary and I've got vegetation and things like that and I have water flowing through there, then the energy expended by that system is going to be truly dependent upon the friction between the water and the boundary.

However, let's say that I have a lot of obstructions. Let's say we put some bridge piers in there, maybe some huge boulders that are not submerged. All of a sudden then, we have energy expenditures due to how the flow has to get around these. And we have something we call form dragging. And we'll talk about that when we get to the momentum equation in Lecture 10. But if we have this form drag and we have all these different eddies and you see, if you've ever been out in the river and you've seen a downstream of--even when you're wading in a river, if you look at downstream where your leg is, you can see eddies and various vortexes that form. This is causing turbulence and eddy diffusivity, and basically you have additional losses or energy expenditures that are not simply due to the friction of the water running over the surface. So, again, you have to be very careful in those kinds of situations that that equation or that relationship between shear stress and the friction slope is no longer valid. So you have to be very careful in those particular cases.

Now, if we return to our force balance equation, we can see, if I repeat it here on the slide, we've got the sum of the forces in the X-direction is equal to the mass times the acceleration of the gravity I--excuse me, times the acceleration of fluid I is equal to the pressure force or the resultant force due to hydrostatic pressure, F1, in the downstream direction plus the weight in the X-direction, so that will be the weight force or the weight of the fluid times the S sub-zero, which is the bed slope, minus the hydrostatic force on the downstream end opposing the flow minus the shear stress or the shear force that's resisting the flow. If you look at that, and we assume that we have steady uniform flow, we know that if we have steady uniform flow, again, our depth is not going to change from upstream to downstream. Therefore, because we know that the pressure is related to the depth, we know that F1, the upstream hydrostatic force, is equal to F2, the downstream hydrostatic force. And because they have opposite sines they'll cancel each other out. We also know that if the flow is steady the acceleration is going to be zero. So we can get rid of that term on the right side of our equation.

If we simplify our equation then, we will note that the bed shear stress is equal to gamma times the cross-sectional area divided by the wetted perimeter times the bed slope, S sub-l. Or we've

got that as gamma times the hydraulic radius times S sub-l. Now, if you'll recall a couple of slides back, we had that that bed shear stress, tau sub-zero, is equal to gamma R times the friction slope, S sub-f. Well, you should note now that if we have steady uniform flow, that the friction slope, S sub-f, is going to equal the bed slope. So many times we will, if we make that assumption in many of our equations, we can approximate the friction slope which we'll use in things like the Manning equation that we're going to be introducing in the next section. We'll actually use the bed slope in its place.

Okay. We have derived the formula for the tractive force and begin to have some understanding of how the slope of the energy grade line or friction slope relates to the bed slope. Knowing this relationship will be important later as we look at some of the assumptions surrounding our use of the energy equation for computation of stream flow using indirect methods as well as when we use steady state, step backwater models such as HEC-RAS.

This concludes this section. In the next section, we will examine the flow resistance equations with the most well-known, here in the United States, being the Manning equation.

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