Development of the Momentum Principle

SPEAKER Rick Huizinga: In this lesson, we're going be discussing what is called the momentum principle. In Lesson 4, we introduced two very useful tools for solving problems in open channels. These were the energy equation and the continuity equation. The continuity equation is Q is equal to volume times area--excuse me--Q is equal to the velocity times the area. The momentum principle will be a third tool that we'll use in our toolbox for solving open channel flow problems. If you can imagine water flowing through a black box at a constant discharge, Q, while the water is in the box, the particles of the water are accelerated or decelerated from V1 to V2.

Now, in Newton's Law, force is equal to mass times acceleration or mass times inertia. In order to change the acceleration of a mass, m, a force, F, must be applied. Well, that force can be written out as "m" times "I", and the inertial force, "I", can be rewritten as the change in velocity over the change in time--delta V over delta T--and that, of course, is written in the equation at the bottom of your screen; mass times V2 minus V1 over delta T

Well, up to now, we've worked with steady uniform flow and open channels and typically, we've assumed that the acceleration of the mass of the fluid "I" is equal to zero so that the sum of the forces equal to "m" times "I" is equal to zero. We determine that all of the forces acting on the mass of fluid and then we set the sum of those forces to zero, such as in Lesson 5 when we said the sum of the forces which were equal to F1 plus the weight times the sine theta minus F2 minus tau LW were equal to zero, and we use that to solve for the tau, the shear force acting on the mass of fluid. But in this lesson, we're stating that the acceleration of the mass of the water is not zero. Acceleration "I" has some finite value other than zero that causes the mass to accelerate or decelerate so that V1 and V2 are different.

Well, this mass of fluid in the box can also be rewritten as the volume times the density. And delta T, which is the time required for the parcel of water to pass through the box, can be rewritten as the volume over the flow rate, which is simply volume over time. So replacing these in our equation for inertia, we can see that the sum of the forces is equal to rho times the volume times V2 minus V1 all over the volume over the discharge. Well, we're gonna move the discharge up to the top since it's in the lower denominator--we can move it to the numerator. And the two volume

terms cancel out, leaving us with the sum of the forces is equal to rho times Q, the discharge, times V2 minus V1. And this is the volume--excuse me, this is the momentum equation for use--this is our third tool to use--to solve open channel flow problems. This is a vector equation since the forces and velocities are vector quantities. They have magnitude and direction.

And as such, this equation can be applied in any of the three coordinate planes using the components of the forces and velocities in each of those coordinate directions. This equation also can be applied to all fluids as long as all of the forces, including the shear forces, are included. There are no limitations to the use of this equation as long as all of these forces are considered. Now, the process is we determine all of the forces acting on the mass of water in the black box, we add those forces together, keeping in mind the direction in which the forces act, and we set the sum of those forces equal to the quantity rho, Q times V2 minus V1.

Some further considerations though: water is seldom flowing into and out of a black box, but this concept of a box or a “control volume” is useful for analyzing these sorts of problems. Pressures and shear forces internal to the box or control volume can be ignored but the forces acting on the outside of the box or the control volume are the forces to be considered when using this tool. This is the end of the lecture portion for Lesson 10. In the next section, we’ll work through an example problem that will help reinforce the concepts of this lesson.