Dimensional Analysis

SPEAKER Bob Holmes: Okay, we're going to talk about dimensional analysis at this point, and this is going to be a little bit of a detour because in some cases this is somewhat hard for students to absorb and even understand exactly why we go this route. But we want to be complete here, and this a subject that, for those of you involved in experimentation, that you would use any time you do a theoretical model study or you're trying to understand something, for example, if you were to look at the drag on a particular vegetation. Let's say that you're going to do a model study where you're going to install wooden sticks to simulate little roughness particles or vegetation of grass or some other woody vegetation, you'd want to know what kind of parameters do you need to collect in your model study that would be meaningful to determine what the drag forces are or what the resistance is of that grass or woody debris, and what kind of impact it has on the flow. This particular lecture on dimensional analysis will give you an idea of how to do that. Some of our fluid mechanics problems can only be solved by experimentation. So, again, we're going to go through this lecture on dimensional analysis to look at how we would ensure we collect the correct parameters.

From a general understanding of fluid phenomenon, one first predicts the physical parameters which influence the flow. And then by grouping the parameters into dimensionless combinations, a better understanding of the flow phenomenon is made possible. That in a nutshell is what dimensional analysis is all about. Most of the time, when you look at experiments, we want to be able to scale up and down. If we do an experiment in the flume, we want to make sure that that flume data that we collect is going to be applicable at riverine level or at a major river level. If it isn't, we need to know that because that particular science that we derive out of there, the particular phenomena or equations that we might come up with or particular laws or axioms or whatever it would be, we don't want to be applying that in a large rivery environment if it doesn't scale to that. So we need to know that, and we often dimensionalize non-dimensional numbers where we can plot them on the grass, where we can look at data collected from a river, data collected on a flume. And if we have the same trend in the data overlay each other in those dimensionless combinations, we have confidence that that phenomena or that physical

understanding of the process that we're trying to relate to is going to be applicable in both those environments.

For example, look at the velocity profiles that I have here on the left. Those are in dimensional terms. On the X-axis, we have the depth in centimeters, and on the Y-axis, we have the maximum velocity minus the velocity at each location depth plotted on the Y-axis. We have the abbreviations for the data. Some of the data is collected in the Missouri River, and those are Mo-2 and Mo-1, and some of the data is collected in a laboratory flume, and that's from [PH] Dr. Lynn in 1993 in his paper and [PH] Bennet Baston in their landmark paper in 1995. And you can see that in dimensional terms, those data plot nowhere near each other. However, if we non-dimensionalize those, in this case, we divide the Y-axis data, the U, the maximum velocity U-max minus the velocity depth ticker point. We divide that by the sheer velocity, which we will derive and explain in later lectures. But we have--those are in dimensionless terms, and we non-dimensionalize the X-axis, the Y over H--with Y, the value of the particular location in the vertical Y, we non-dimensionalize that with the value of the total depth H, we can see that those data collapse on each other and we have a fairly firm understanding that--we're confident that we have the phenomena that we predict here and how the velocities relate to the flow depths that is applicable both in the Missouri River and in the flume situation that we were collecting the data in.

Okay. We can express most variables in fluid mechanics in the following three dimensions, and we covered this in Lesson 1. We have force, which is abbreviated with capital F. We have time, which is abbreviated with a capital T. And we have length, abbreviated with capital L. For clarity in this lecture, when I have the brackets, which is the bracket terms--left bracket and right bracket--I mean the dimensions of. Now, for pressure, we would state pressure in brackets. It has the dimensions of F over L squared. Now, a rational equation is one which is developed from physical laws. It must have balance in magnitude and dimension--and be dimensionally homogenous. That is, the dimensions on the right-hand side of the equation must be equivalent to the dimensions on the left side of the equation, and each term on the equation must have the same dimensions. The question becomes: How do we know how to group our variables into something coherent for us to use in an experiment? Well, we get to something we call the Buckingham pi

theorem. The Buckingham pi theorem is basically where we look at the number of the independent dimensionless groups, that's abbreviated with a capital G, is equal to K minus R, where K is the number of variables that we have and R is the minimum number of basic dimensions. Let me explain. If I have example of where I'm looking for the drag force, I look at the physical concept of that and I say, "All right, the velocity V, the density of the fluid, the dynamic viscosity of the fluid mu, and the diameter of the item that I'm placing in there such as a bridge pier, those are the important components. By inspection of those components, I see that I have three basic dimensions involved. I have the length dimension. I have the force dimension, and I have the time dimension. I also know that the total number of variables is five. Note, we count the drag force F sub-D, so the five variables will be the drag force, the velocity, V; the density, rho; the dynamic viscosity, mu; and the depth--or the diameter of the pier, D. Now, if I apply the Buckingham pi theorem, I've got G is equal to K minus R, right? So G is the number of independent dimensionless groups. If I have five parameters, as I have just discussed, and I have three basic dimensions, I have five minus three, or two independent and dimensionless groupings for us to use in correlating this experimental data.

Now, we have some rules for dimensional analysis, and those are listed on your screen. Number 1, we want to list all the independent variables that are involved in the problem and write some sort of functional equation such as Z is equal to A, B, C, D, where Z is the dependent variable. In our previous example, that would be the drag force. Now, we want to examine appropriate physical laws to help us select the correct parameters. That's how we get the A, B, C and D. So if you get out your physics book and you've looked at your fluid mechanics and hydraulics literature, you can look at the problem you're trying to solve and come up with a list of things that might be important and we would list those in this equation as A, B, C, D, E, et cetera. Now, we want to make sure all of those variables are independent. For example, you wouldn't want pipe diameter and the cross-sectional area as a variable because they are not independent. Cross-sectional area is a function of the pipe diameter. We also--the last thing we want to do we want to keep this number of parameters as minimal as possible. Now, the next step we would do is express each variable in terms of the basic dimensions. That's easy enough. We would take each

of our independent parameters. A, we would look at the dimensions that it has. We list those. We'd move on to the next parameter B, C, D and so forth.

Next, we would use the Buckingham pi theorem to determine the required number of pi terms or dimensionless combinations of the variables. Next, we'll be selecting the dimension that repeats quite a bit and seek to eliminate it by taking a variable that includes that dimension and divide through by the variable of sum power of the variable--or sum power of the variable. Now, don't choose the dependent variable Z, as we only want it to show up in one pi term. That's on the left-hand side of the equation. By inspection, we combine all other variables that contain the dimensions in such a way as new terms do not contain that dimension. We can--and subsequently, we repeat that step for until we have dimensionless terms throughout. Now, we're gonna violate--for those of you who are very familiar with algebra, we're going to violate the algebra concepts because we're going to divide through and get rid of terms, but we're not going to necessarily divide every term by that particular value.

We have some useful hints. We're going to eliminate the dimensions of force first. If rho is available, rho has, which is the density of the fluid, rho has force in it as a dimension. So we're going to combine that rho in an appropriate manner with each variable containing the force dimension to eliminate that dimension. For example, the rho or the density value has dimensions of force times the time variable or time dimension squared divided by the length dimension to the fourth power. Next, if the velocity is present, we want to get rid of the time dimension by using the velocity. Now, if only one variable contains the time dimension, then add the acceleration of gravity to the list of variables because it's usually part of the driving mechanism of flow, particularly open channel flows.

Next, if depth is available as a variable, use it to eliminate the length dimension. After all terms are dimensionless, it is perfectly legal in our dimensional analysis sense to take terms to any power or invert them.

That concludes this part of the dimensional analysis lecture. The next section will contain an example of how to work these kinds of problems. Bed shear stress