Example Problem Energy Equation

SPEAKER: Okay. Let's work an example problem here where you will better understand how to solve the problems that are given to you in the book, and we're going to use both the energy equation and the continuity equation to solve this. Now on your screen, you have a diagram and basically it's a free-body diagram where I'm showing a stream that's flowing from left to right and you have some known depth upstream and you know what the flow rate is. We know what the distance of the bed elevation is above the datum at both locations, and we are asked to solve for what the depth is downstream and the velocities at Section A and B.

Now, in all these problems that you're going to work, I always encourage people to not only draw out the free-body diagram but write down everything you know, and then write down your tools, the tools being the various equations and concepts that we've learned up to this point.

In your handouts, you should have an equation sheet. So look for the equations that we have in Lessons 1 through 4, and you'll see all those and you'll know exactly the tools that you have to work these particular problems.

Now in regards to the things which are known as part of this problem--I know that my flow rate is 280 cubic feet per second or cfs. I know my depth at Section A, D sub-a, is five feet. Again, this is where I'm using D to designate the depth as opposed to Y. Remember, in the actual lecture section where I derived the energy equation, I used the value Y sub A. In this particular case, I'm using D sub-a so do not get confused. This is just the depth above the bed, the water surface elevation. This is the water surface elevation above the bed, D sub-a. I know that the width in this particular location at Section A, B sub-a, is equal to eight feet. If I move downstream, I can see that the distance between the datum and the bottom of the channel or the bed of the river at Section B, that is designated Z sub-b is equal to one foot. I have no idea what the depth is there, nor do I know what the velocity at that particular section is. I am given that the width of the channel at Section B is 10 feet.

Now, I start to write down my equations. Well, I have the energy equation, which is the VA squared over two G plus DA plus ZA is equal to the velocity at the downstream section, D sub-b, that value squared divided by two G plus the depth at Section B, which is D sub-b, plus the

elevation of the bed above the datum, which is Z sub-b, and that's equal to a constant. We also have the continuity equation. That's A sub-a times the velocity at Section A, V sub-a, is equal to V sub-b, or the velocity at Section B, times the cross-sectional area at location B, A sub-b, that's equal to the volumetric flow rate, Q, and again, that's in cubic feet per second.

Now, I can solve for the value of the velocity at A simply by using the continuity equation. If I know the continuity equation says that the area of a cross-section at Section A times the velocity at Section A is equal to the volumetric flow rate, Q, I can algebraically manipulate this equation where I plug in the value of my knowns. I know that the area at Section A is equal to the depth, five feet, times the width of the cross-section there, which is eight feet--we're assuming this is a rectangular channel--multiplied by the velocity at Section A, V sub-a, which we don't know yet but we're going to solve for that, is equal to the volumetric flow rate 280 cubic feet per second. We manipulate that equation and we can solve for the velocity at Section A, and we see that that's equal to seven feet per second. So now one of my three unknowns is now known.

Now, I've solved for everything at Section A. So I know the velocity so I can solve for the velocity head, V squared over two G. I know the depth so I now have the pressure potential head. And I know the value of the potential energy which is my distance above the datum. If I move downstream to Section B, I see that I still don't know what my velocity at Section B is, nor do I know what the depth at Section B. So I pull in my energy equation and start to solve for things at Section A.

All right, so if I write down my energy equation, the first thing I wanna do is I wanna solve everything on the left side of my energy equation, because that would give me my total head value. That would be at Section A. Because I know everything up at Section A, so I know what the constant is going to be for this energy. So if I solve for that, I plug in the value for VA, I plug in the seven feet per second that I know we solved for the velocity at Section A. I plug in the depth, I plug in the potential energy, and I can get that the total energy at Section A is 8.011 feet.

Now, if I am still using the energy equation, I can plug in the value of this, 8.011 feet, on the left side of the equation and then I plug in everything on the right side of the equation that I know. I

know that the velocity at B is still unknown, so I leave that alone. But I square that and divide it by two times 32.2 feet per second squared, which is the acceleration of gravity. I add to that the value of the depth at Section B. I don't know that. So we'll leave that as D sub-b. I'm going to abbreviate that, and I'm going to add the value of one, which is the value of the potential energy above the datum at Section B.

Now, I have a problem. I've got this energy equation, but I have two unknowns. So for those of you familiar with algebra, you remember that if you have two unknowns, you have to have two equations to solve for it. So what I'm going to do is I'm going to bring back in my continuity equation, and I'm going to say, "All right." If I know that the cross-sectional area B times the velocity B is equal to the discharge of some known value, Q, I can manipulate that and say that the velocity B is equal to the volumetric discharge of 280 cfs divided by the cross-sectional area, which is D sub-b times the width, which is 10 feet. Or I can say that VB is equal to 2.8 divided by the depth at Section B.

Now, what I'm going to do is I'm going to take that value and I'm going to substitute that back in to my energy equation previously in place of V. So instead of writing VB squared, I'm going to now write 2.8 over DB, and I'm going to square that value. Do you see how that gets substituted into that equation? Now, I have one equation and one unknown. But it's a cubic equation, so I'm going to have a little bit of a problem solving that, unless I use a trial and error process, which is what we're going to do. I'm going to--as you can see on your screen, I further manipulated the equation to where I've got it in terms of 7.01 feet on the left side. I just basically taken the one foot that I had for a potential energy on the right side and I subtracted both sides by one so I get 7.011 feet on the left side, and I've got 12.74 divided by the value of the depth squared plus DB. Now, what do we do with this? Well, if you have a sophisticated calculator, some of the calculators these days that students have, you can solve this. You can just enter this in and run solver on it and it will solve the value for you. If you have a spreadsheet, you can plug these kinds of equations in and use the add-in solver in your Excell spreadsheet or whatever spreadsheet you use to solve for that. But I'm going to assume that you don't have that. So we're going to have to use a trial and

error solution, which is what I would prefer that you use for working your numbers out as you work through these kinds of problems, because that gives you a feel for things.

The problem with computers and automated computer models and things like that, they are so user-friendly that often people that really don't know what the dynamics, they treat it like a black box and they don't understand how the pieces fit together. So in this particular set of problems that you're going to work, whenever you work through these, I prefer that at least you work one of these particular problems by trial and error. That gives you kind of a feel for how things come together.

Okay, so I'm going to--I rewrite the equation at the top of this slide, which is 7.011 feet equals to 12.174 divided by the depth at Section B squared plus the depth at Section B. Now, as we've said, this is a problem that you could solve using your calculator, if you've got the solver solutions technique on there, if it's a sophisticated enough calculator, or you could use some numerical method like a Newton-Raphson. But in this case we want you to use trial and error. That always, even with simple calculators that we all might have, we can always solve it.

What we're going to do is we're going to plug in various values of D sub-b on the right-hand side of the equation, and we're going to iterate through various values until we get the right-hand side of the equation to equal to the left-hand side or the 7.011 feet. So you can see on your table, I plug in a value of 10 feet in for an assumed value of the depth at Section B. Again, that's not right, we're just guessing. And I can calculate, run it through there into the right-hand side of the equation, square it, divided into 12.174. Add that value at 10 feet again, and I come up with a value on the right-hand side of the equation of 10.12 feet. Well, obviously, that doesn't match 7.011 feet. So now I'm going to guess another value. So I go down a little bit and I guess a value of eight. I get a value of 8.19 when I do that. I plug that value of eight feet into the right-hand side of the equation. That's not right so I need to iterate again. I make a guess of seven feet. I get a value of 7.248 feet. Again, that's that not correct, but I'm getting closer. They're getting a little closer to what the left-hand side is. I go down to 6.75. I know that the right-hand side of the equation now is 7.017. It's not quite exactly 7.011, but it's close. I go down 6.74 and I see that

now I'm below that estimate--or the value in the left-hand side, I've got a value of 7.008. We know those two values, 6.74 and 6.75, bound the true value on the left side.

Now, a property of this kind of cubic equations are that there are two values, two roots that would satisfy this equation. So it's beyond what you know at this point in your skills that you've attained in this particular course. But as we move on, we're going to have the concept of alternate depths, and we're going to see that there are two values of depth that satisfy the specific energy equation, which is what we've got in front of us here. We haven't called it that at this point, but that's essentially what it is when we are looking at the potential energy plus the depth. I mean, I'm sorry, the kinetic energy plus the depth.

So we see that if I switch over to the right-hand side of that table, if I assume a value of two, I get a value for the right-hand side equal to 5.043 and--that's not it. So I'm now looking for that second root of this equation, the second value of D sub-b that will make the right-hand side equal to 7.011. I iterate down and I see that between a value of 1.49 and 1.48 feet, I get a value of 7.011. If I plug those values, 1.49 into the right-hand side for DB, I get the value of 6.973. If I plug in the value of 1.48, I get a value of 7.038. And so I know that those two values bound my actual number, the value that will equal 7.011 when I plug it in to the right-hand side of the equation. Now, we come up against the issue of which one of these roots is the correct value to use. We'll talk about this later. But essentially because upstream I had a depth of five feet, I know that my value between 6.74 and 6.75 is an unlikely value, unless I have some kind of a backwater situation that will back water up. Because that's not the case, we're going to pick the value of 1.4 and we're going to interpolate between 1.48 and 1.49, and we're going to call it 1.484 feet is the actual depth that we're going to use for our problem, for the depth at Section B.

Now, I go back to my continuity equation and I say using that computed value of 1.484 feet, if I multiply that times the width of the channel at Section B, which is 10 feet, divide that into my volumetric flow rate, which is 280 cubic feet per second, I get a velocity of 18.87 feet per second is the velocity at B. So we've solved for the velocity at section A. We've solved for the depth of Section B, and now we've solved for the velocity of Section B. So we've completed the problem.

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We've solved for all the unknowns.

I hope at this point you go into your Lesson 4 and work the problems that are there on the back of that particular lesson.