Example Problem Flow Resistance

SPEAKER Bob Holmes: Well, we're going to work an example problem now to show you the application of using the flow resistance equation, particularly the Manning equation, and we're going to assume we have an open channel uniform flow. And before I get to that, I'm going to show you a table that's out of Ven-Te Chow's 1959 Open Channel Hydraulics textbook. This will be fairly useful for you when you have standard cross section shapes. This gives all the equations for area, wetted perimeter, hydraulic radius, hydraulic depth, which is something we haven't talked about yet, but that's basically the area divided by the top width. And so you can look through and see--it's on your screen now--and you can see that for a cross-section that's rectangular, or for something that's a trapezoidal shape or triangular, circular, et cetera, you can see that for these various shapes you can examine the table and get the equation for the particular quantity you're looking for. If it's area, you look in the first column, and you can see what the actual equation is based on the geometric elements of that particular shape.

All right. So let's set up the problem. We have a trapezoidal channel that has a base width of 10 feet on the bottom. It has a two-to-one side slope on each side. We're given that the flow rate is 4000 cubic feet per second. We're given that the bed slope is 0.002. Now, you know that if this is a uniform flow, we've already derived in the previous section, that the friction slope is equal to the bed slope, so that is equal to 0.002, and that would be dimensionless. It's feet per feet, and that the N value is 0.013. Now, we're going to get into actual N values, but I can tell you right now this N value is very low. This is a very efficient cross-section which means that the roughness particles along the slope or along the channel bed is very, very smooth. This would be a very slick, concrete or fine plexiglass and that neighborhood. Typically, in natural channels, we're going to be upwards of 0.029 and above, about 0.035. And there are various manuals that the U.S. Geological Survey has put out for actual pictures and where we've done roughness studies, where we've measured the flow rate, measured all the characteristics of the channel and the water surface slope and we back-calculate the N value. So you can actually see what a standard N value looks like.

For this problem, though, we are using 0.013. Now, we're given that the Manning equation is the discharge, Q, is equal to 1.49 over the roughness coefficient N times the cross-sectional area A times the hydraulic radius to the two-thirds power times the slope of the friction to the one-half power. Now, again, we're going to use the bed slope for that S sub-f. So if I start plugging in the things that I have, I can see that I know everything but the area and the hydraulic radius. Obviously, we're asked here to calculate the water depth, the flow depth, so we're going to have to do some work to get there. After we simplify this through, we get that the value of the 4000 cubic feet per second is equal to 5.13 times the area times the hydraulic radius to the two-thirds.

Now, you can see to the lower right of your screen that the area is going to be 10 times the depth plus two times the depth squared. That's out of the chart or the table that I gave you out of Ven-Te Chow's 1959 Open Channel Hydraulics textbook. The wetted perimeter for a trapezoidal channel is going to be 10, which is again the base width, plus two times the depth to the square root of five. Now, again, that five comes from the one plus Z squared, which comes out of the table, and the Z being the value of the horizontal component of the side slope. Now, you can see that this value of D shows up in various forms. It's there to the first power and it's also there in the square, and so it's going to be very difficult to solve directly for the D out of the equation. Therefore, we're going to resort back to a trial and error solution.

Now, again, if you have a sophisticated calculator or you have--you wanna use the solver function within the spreadsheet, you can do that, but we're going to use a trial and error method. So you can see now that I've set up a table where I've got the depth listed on the left in the first column and then I've got a column for A, which we can calculate independently. I can calculate the wetted perimeter if I know what the assumed depth is and then obviously I can divide the area divided by the cross--the wetted perimeter to get the hydraulic radius, and then lastly I can compute the discharge, Q, at the right end. Now, what we're doing here is we're--we know that the Q is 4000 cfs, and then we've got the right-hand side of the equation, that's a lot of terms, everything is in terms of D, but it's D to the square to the two-thirds power, et cetera. I mean, there are all kinds of different gyrations of D there. So we're going to go through and assume a D

and then we're going to calculate all the geometric elements, the area, the wetted perimeter, the hydraulic radius, and then we're going to plug those into that Manning equation and calculate what the flow rate is. And when we get that flow rate to match 4000 cfs, we have derived our answer. All right?

So the first thing we're going to do is we're going to assume a value of six. We calculate the cross-sectional area at 132 square feet. Then we compute the wetted perimeter at 36.83 feet. We divide those two and get the hydraulic radius. We take the two-thirds power of that. We plug all those values in the area and the hydraulic radius and we calculate a discharge of 1585 cubic feet per second. Obviously, well, that's not 4000 cfs so we need to move up. Now, in order to get the value of Q to go up, we're going to have to increase the depth. So we do that. We assume a value of eight. We go through. You can see the table is populated there are 208 cubic square feet for the area, 45.78 feet for the wetted perimeter, and we come out there to the right, we have a value of the discharge based on an assumed depth of eight feet of 2925.

Skipping further down the table, you can see that if I assume a value of 9.2 feet, I compute the various values of area, wetted perimeter and hydraulic radius, and I come up with 3973. I'm almost there. I jump up to 9.3 feet. I calculate a discharge of 4068 cfs. I know that my value of depth is somewhere bounded by 9.2 and 9.3 feet. Instead of going through relatively on every hundredth of a foot, I'm just going to interpolate between 9.2 and 9.3, and we're going to see that that value of depth for a 4000 cfs flow through that channel for that particular roughness at that value of slope is 9.23 feet.

That concludes this example problem. I now encourage you to go to the textbook and work those problems at the end of this lesson.