Example Problem Computations

SPEAKER Rick Huizinga: Let's work through an example problem for lesson 15, in water surface profile computations. I want to reiterate here that there are computer programs that take care of this sort of thing for you nowadays such as HEC-RAS or WSPRO or any number of other computer models that will do this sort of thing automatically. But it's important to understand what the computer is doing because trust me, the classic saying of, "Garbage in, garbage out" really applies to water surface elevations, and you can get some absolute nonsense out of those computer programs if you don't understand what you're doing with them. Well, in our example problem, we're given the fact--we know we have a 100-foot wide rectangular channel, that it has a slope of 0.0006 feet per foot, that it has a Manning's n of 0.03, that is has a discharge of 1669.2 cubic feet per second, and that it has an observed depth of 4.5 feet. Now, we're going to make a few assumptions as well. We're going to assume that our Coriolis coefficient, alpha, that they're all equal to one, and we're also going to assume that our conveyance and contraction losses are also equal to zero. So we don't have to worry about conveyance and expansions and contraction losses.

They want us to determine the depth 300 feet upstream from the known depth of 4.5 feet. But first of all, we need to determine what type of flow we have by computing the Froude number. And if we say that velocity is equal to Q over A, which we know that that's true from continuity, we can compute what the velocity is. Our discharge 1669.2 over the area, which would be the depth 4.5, times the width of the channel 100, that gives us a velocity of 3.71 feet per second. We plug that into our Froude number equation, which is V over the square root of gD, we see that we can insert our value of 3.71 for velocity, we know that g is 32.2 and the depth is 4.5. That gives us a Froude number of 0.31. Well, since that Froude number 0.31 is less than 1, we know that flow is subcritical, which means that we have a mild slope and the control is at the downstream end and computations will proceed upstream, which is good because they ask us to find the depth 300 feet upstream from our current known depth. Well, let's compute the normal depth then and we'll assume that the friction slope is equal to the bed slope. We'll use our Manning's equation, Q is equal to 1.49 over n times A times R to the 2/3 times the square root of S sub f. So we plug-in our knowns, 1.49 over 0.03 times 100 times D normal, that would be the area, times the quantity 100

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Dn over 100 plus 2 Dn taken to the 2/3 power, that's our hydraulic radius to the 2/3, times the square root of 0.0006. When we solve for Q, you then get that equation--that's in the middle of your screen, 1.22 times 100 Dn times the quantity 100 Dn over 10 plus 2 Dn all to the 2/3 power.

We have to solve that by trial and error, and so we'll start with a normal depth that's equal to our known depth. We say that Dn is equal to 4.5, you get a Q of 1408.9. Well that's not equal to the discharge of the channel. So we increased our depths until we find that at a depth of five feet, the discharge is equal to 1669.2 cubic feet per second. So our normal depth then is five feet and it's also greater than our known depth, which we already know is greater than critical depth, and so we are going to be in the area between critical and normal, which means we're going to be following an M2 profile. And the normal depth is greater than the depth 300 feet upstream, which is greater than the known depth. So the upstream depth will be between 4.5 and 5 feet.

Well, let's compute some parameters at the downstream section. Let's compute the conveyance. Start first of all, conveyance using the manning equation is 1.49 over n times A times R to the 2/3, so that's equal to 1.49 over 0.03 times 100 times 4.5 times the quantity 100 times 4.5 over 100 plus two times 4.5 all raised to the 2/3 power. So we get a value of conveyance of 57,518 cubic feet per second. We can then compute the friction slope at the downstream end and take--we will compute that as the Q over the conveyance. So we enter our Q and we enter our conveyance values and we find the friction slope at the downstream end of 0.000842. We compute the velocity at the downstream end as Q over A. We enter our values there and we get our velocity of 3.71 feet. We computed that earlier actually. And then we can compute our velocity head, alpha V squared over 2g, assuming that alpha is one, we enter our V and 2g value and we find that our velocity head is 0.214 feet. We're gonna put these results in the table and we're gonna start on the next upstream section.

So in front of you, you see the table that we've created. When we insert our parameters, you can see what we have there. We're going to compute parameters at the next upstream section and we're going to assume that the depth of that next upstream section is 4.7 feet. We need to compute the rise in the bed between the downstream and upstream sections and we see that the Z sub u is equal to the bed slope S sub zero times the length. So 0.0006 times 300 feet gives us

a rise in the bed of 0.18 feet. So the water surface elevation is equal to the depth plus the Z at the upstream section of the reach. So 4.7 plus 0.18 gives us an overall water surface elevation at the upstream end of 4.88. And this is an assumed upstream water surface elevation.

Well, let's compute the additional parameters at the upstream section using those values and being careful to use the depth at the upstream section not the water surface elevation in the equations, all right? So our conveyance at the upstream section, 1.49 over n times A times R to the 2/3, when we insert our values into that, we get 1.49 over 0.03 times 100 times 4.7 all times the quantity 100 times 4.7 over 100 plus 2 times 4.7, all raised to the 2/3 power. That gives us a value of conveyance at the upstream end of 61,690.

The friction slope at the upstream end is again Q over K squared. We put our values in there and we get 0.000732. We compute the velocity and we see that that's 3.55 and we compute our velocity head and we see that that is 0.196 feet. Now, we need to determine the friction slope and we're going to use the geometric mean to do that. The friction slope--the average friction slope, S sub f bar is equal to the square root of the friction slope at the downstream end times the friction slope at the upstream end. We insert those values and we get an average friction slope of 0.000785. Our friction loss then is the average friction slope times the length of the reach, 0.000785 times 300, gives us the friction loss of 0.236 feet. We compute the water surface at the upstream section then using the energy equation, and we plug-in the various values that we have. For H sub d, the velocity head at the upstream, the velocity head at the downstream end, the friction losses--we don't have any eddy losses. We find that our upstream water surface elevation should be 4.75 feet. But when we insert those things into our table, we find--and I'm just going to show you that in the table itself, we find that 4.75 does not equal the 4.88 that we assumed. So our next step is to take the computed water surface elevation and use that as our new assumed water surface elevation.

We have to now work backwards to find our depth. We will take the upstream water surface elevation and subtract off the Z in the upstream to get our new upstream depth. And we see there at the equation at the bottom of your screen, the new depth upstream is the assumed water surface elevation of 4.75, we subtract off the Z of 0.18 and we get a new upstream depth of 4.57.

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And we now are going to re-compute our parameters at the upstream section, being careful to use that new upstream depth in our equations. And you can see those as we stepped through the equations on your screen there, the conveyance is now computed to be 58,966. The new friction slope is 0.000801. The new velocity is 3.65. The new velocity head is 0.207. When we determine the geometric mean, this time we have to make sure that we use the downstream value and not the previously determined upstream value. So you need to ignore that intermediate upstream value that we had just computed. So we need to use 0.000842 and 0.000801. Our new average friction slope then is 0.000821.

So the friction loss, multiplying that by the length of--multiplying our average friction loss by the length of the channel, we get a friction loss of 0.246 and we can compute the water surface elevation at the upstream section using our modified energy equation, and you see that it is now 4.75, which checks out when we insert our values: 4.75 is the assumed value, 4.75 is the computed value, therefore, we know that the depth in the upstream section is 4.57. And we do some additional checks. We know that the upstream depth has to be greater than the downstream depth since it's an M2 profile, that checks out. We also know that because it's an M2 profile that it's between critical depth and normal depth, and that also checks out: 4.57 is between critical depth and normal depth, and we can do final check and that is to check the conveyance change from section one to section two. And you do that by taking the absolute value of the downstream conveyance minus the upstream conveyance over the downstream conveyance and we see that that gives us a value of 0.025 or 2.5% change. That's a totally acceptable value. If that change is greater than 20%, then you would want to reduce your reach length so that the conveyance change is not as great.

Now, if we wanted to go another 300 feet upstream, we could assume an additional U sub d, and I'd do that quickly here for you. If we were to assume an upstream depth of 4.7 feet, we could compute the rise in the bed. Now, we would have to look at the Z value being twice as big as it was in the previous section. So Z-u now is S sub zero times 2 times the length or 0.36 feet, and the new water surface elevation would be the assumed depth plus Z or 5.06 feet, and that becomes our first assumed upstream water surface elevation. If you were to compute the

parameters at the new upstream section the same as what we did earlier in line two, you determine the geometric mean and the friction losses. You can see those values there on your screen. If you compute the water surface elevation at the upstream section, you'll find it to be 4.99 feet. Of course, this does not compare very well with the value of 5.06 that we had assumed, and so we would step through the problem one more time assuming 4.99 as our new upstream water surface elevation, back-calculating the upstream depth, resolving all of the equations and you find that after the second iteration, we get 4.99 as the assumed and the computed. Using this procedure then, you see how you could work your way upstream in increments of 300 feet until you would reach normal depth if you wanted to. And generally speaking using this technique, you will resolve that the correct answer within two or, at the most, three iterations as long as you're choosing reasonable values for your assumed water surface elevations or depths. This is the end of the example problem for lesson 15.