Example Problem Computing Discharge

SPEAKER Charles Berenbrock: In this section, we're gonna provide a practical application to compute discharge. As we earlier derived the equation, the energy equation between section 1 and 3, you can see the resulting equation where Q, the discharge squared, is equal to a whole bunch of terms here. There are seven unknowns in this. Some of those unknowns are very hard to derive or to calculate. Mathai provided a procedure or methodology that will help us in applying this equation.

In the first part, Mathai assumed that the entrance losses can be estimated by the coefficient for energy. A loss is due to the expansion times alpha three (α_3), your velocity at section 3 squared divided by 2g. Now this assumption is pretty reasonable because the velocity at three is about the same velocity at section 2A and also the velocity at 2 is much, much less than the velocity at 2A. Then also, if we replace the velocity at three by the continuity equation, you can see the results in equation where a--I mean where discharge now is equal to the area at section 3 divided by the square root of the alpha sub three times the quantity of 1 plus k sub e. And then all of that is multiplied by the square root of 2g times this delta h plus alpha 1 times velocity at 1 squared divided by 2g minus your friction loss between 1 and 3. Note that delta h is really h sub 1 minus h sub 3.

Mathai further refined this equation by setting "C", and C is the discharge coefficient, and you can see what C equals. It equals 1 divided by the square root of alpha sub 3 times the quantity of 1 plus k sub e. Now if we insert "C" into this equation, it yields Q is equal to C times a sub 3 and then there's quantity that has a square root to it. So now, how do we use this equation to compute discharge? Well, Mathai set it up so that so that this discharge coefficient is probably a function of the abutment type, the bridge length, the approach flow angle and the channel contraction ratio and we're gonna go through some of those terms there. Other factors which can affect the "C" value include the Froude number, other contracted section, if there's wing walls or entrance surrounding to the abutments, if there's spur dikes in the reach, that there's piles or piers to the bridge or the bridge is submerged.

This is example of abutment types. There are four types of abutments. You can see that the first type [Type 1] has it without wing walls or with wing walls. Type two is where you have slope on

the upstream side and downstream side of the embankment. Type 3, you also have the slope on the upstream and downstream of the embankment but also, it has some rounding to it. Then Type 4 is where it's like squared rounding at the abutments.

The channel contracted ratio "m" is basically the percentage of flow blocked by the embankment. And "m" is equal to Q minus little q with the prime (q') on it divided by a big Q. Again, this is the percentage of flow block by the embankment. But we really don't know what Q or little q prime (q') is. But we do know that Q is equal to K times S to the one-half power. And remember from earlier lectures that K is your conveyance. So if we substitute K into that equation for Q, we see that "m" is equal to K minus k subprime (k') divided by K again. Now k subprime (k') is the conveyance in your contracted section and large K is the conveyance in your main section.

The discharge coefficient C is really a function of many things as we saw and we see that it's—and so it's based on bridge geometry and your channel contraction. And so C is really equal to C prime (C') times these other factors like k sub F (k_F) times k sub r (k_r), k sub ϕ (k_ϕ) and k sub w (k_w). If one or a couple of those values are not included, you do not include it those in that equation to calculate C. Mathai also included some graphs. He provided those in a report called "TWRI 3A4". TWRI means "Techniques of Water Resources Investigations" and its in report 3A4. These are some of the graphs from that report and you can see that C prime is basically a function of "m", of the conveyance there. And so if you know what "m" is, you can go up and then determine what C subprime (C') is. And then he also provided some graphs of k sub f (k_F). K sub f is a function of the Froude number so you have to determine what the Froude number is. Remember that the Froude number is the velocity divided by the square root of g times some characteristic length [or characteristic depth such as hydraulic depth]. And then also he provided a graph, the lowest graph there, k sub F (k_F) and this is a ratio of corners rounding to width of opening. If you have rounded quarters on the embankment, you would include the k sub F (k_F) term and the equation for calculating "C", your discharge coefficient.

The next graphs also have a few more terms in them like k sub w (k_w) and then K subϕ ($k\phi$). If you had those items in there, you would include those in the equation for calculating C. So we can

see that we can use—we can determine what C is and so Mathai's procedure really simplifies things for us to carefully [determine] what the discharge is through the contracted opening.

The following are steps to compute Q and this is a trial by air procedure. So the first step is to compute K, your main conveyance and your main channel section. So you would calculate it at the approach section, then you compute alpha sub 1 (α_1). The third step is to compute k prime (k') for the contracted section so you can compute the conveyance and the contracted section. From that, you could compute "m", your channel contraction ratio. You should also compute your length to width ratio, that's called L sub b (L/b). So it's the length of the bridge (L) to the width of the bridge (b), then you can determine what C prime is (C').

Step 6 says that you should assume a value for Q. So just assume a value for discharge and then you can compute what your velocity head is, what your friction losses from section 1 to 2, your friction losses from section 2 to 3, your velocity at section 3, the Froude number at section 3 and remember that the Froude number is V divided by the square root of g times the depth at section 3. Then also, you determine all the other k's. Remember we determined the k's from the previous graphs, for example, like k_w or $k\phi$. By then, you can compute your final "C". Once you have "C" computed, then you can go to Step 8 and then compute the discharge from this equation. Remember that delta h (Δh) is h sub 3 (h_3) minus h sub 1 (h_1). And then if the Q and that you computed in Step 8, if it does not equal the Q that you assumed in Step 6, then you need to go back to Step 6 and then let your computed Q from Step 8 equal your new computed Q for Step 6. You repeat this until the Q in Step 8 equals the Q in Step 6. By using this procedure, you can compute the discharge for a contracted opening. You use this to determine--you use this procedure for your example. Now we have a flume demonstration for the rapidly varied flow at constrictions.