Example Problem Lesson 6: Energy Equation

SPEAKER Bob Holmes: Okay. It's time to work an example. We were discussing using the energy equation in a real fluid. Now, on your diagram, we have an example of a bridge constriction and we have a both a side view and a plan view. You'll note that in the plan view you see that the cross-section of the channel is 70 feet wide and then we get into the constriction section, where the bridge is, it's only 40 feet wide. The flow is from left to right in your diagram. You'll note that I've also drawn the water surface as it flows from left to right from Section 1, which is the approach cross-section, upstream of the bridge. We have the upstream face of the bridge, which is Section 2, and the downstream face of the bridge, which is Section 3.

Now, what you should do is roughly sketch on any problem you're given the energy grade line and label all the known and unknown values. We note in the problem that we know that the depth upstream is 6.5 feet. That's the depth from the water surface down to the bed of the river. Now, we also know that we've got a datum that we've said here where we have zero value--the value of zero for the potential energy at Section 3 and the potential energy at Section 1 is 0.34 feet. We don't have to have that datum really low. Here we've got a zero set at the bed elevation at the downstream end of the bridge. We don't know what the velocity is through each of the sections, but we're given that the head loss from Section 1 to 2, which would be from the approach cross-section to the upstream face of the bridge, that has a value of 0.25. So we call that H sub-l and then superscript 1-2. That denotes that we have the head loss from Section 1 to 2.

The head loss from Section 2 to 3 is denoted H sub-l with a superscript 2-3, which tells us that's the head loss from Section 2 to 3. We're given in the problem that that value is 0.35 feet. We also are told in the problem that the value of the flow depth at Section 3 is five feet. Now this is fairly common for an indirect. Those of you that may know something about this, we'll talk about it later in one particular lesson toward the end of this course, but this is some of kinds of data that you would have, the knowns that you would have when you have a contracted opening, sort of as a bridge, where we go out--where we wanna go out and determine what the flow discharge is when we aren't able to get a discharge measurement. We would go out and survey the cross-sections and then the bridge opening, and we would determine how high the water surface was by looking at high watermarks, and so we would know the flow depths and we would know the datum, the

potential energy and the pressure potential energy for each one of those sections. Again, we don't know what the velocity is. We don't know what the discharge is. We need to solve for those.

Now, again, we always wanna write down our tools. Our main tool in this particular problem is going to be the energy equation for a real fluid. And you can see again that we've got the alpha values on each of the velocity heads both at Section 1 and Section 3. We have the head loss from Section 1 to 2 and from 2 to 3. Now we usually break those apart because there are gonna be various--when we get into the indirect, actually the application of this, you're gonna find that when we calculate the head losses, it's gonna be dependent on the cross-section and the roughness. So, typically, in a bridge opening, we have different roughness through that bridge opening that we would have from the approach to the front of the opening because usually we've got vegetation, things on the banks, whereas in the bridge opening or through the constriction you might have concrete wing walls.

Now, if we substitute in the energy equation all the values that we know, we've been given that the alpha one value, the energy coefficient value for Section 1, is 1.45. Recall, I told you earlier, that it's always a value greater than one. Sometimes we'll just assume it's one if we don't know any better, but we always go through the computation. So we put in the 1.45 for alpha. We know that the acceleration of gravity in British gravitational units is 32.2 feet per second squared. So if I multiply two times 32.2, I substitute in 64.4 and the denominator of the first term. We also know that the depth in Section 1 is 6.5. We know that the potential energy or the height of the bed above the common datum is 0.34 feet. On the right side of the equation, we're given that the energy coefficient is 1.08. The velocity in that Section 3, we don't know what that is so we'll leave that alone for the time being and we'll leave it squared. Again, we know that the acceleration of gravity is 32.2, so we multiply the two by 32.2 and get 64.4 for the denominator in the first term on the right side of the equation.

We're also--we were given that the depth was five feet and that the datum coincided with the bed at Section 3 so we enter a value for zero. The head loss term from one to two and the head loss term from two to three, we'll plug those in at 0.25 and 0.35 respectively. We go through a simplification of the math on both sides of the equation and we come up with the term on the

bottom of your screen, which is 0.225 times the velocity at Section 1 squared plus 6.84. In the right-hand side of the equation is 0.0168 times the velocity at Section 3 squared plus 5.60.

Now we have one equation and two unknowns, so we have to find another equation. So, as you guessed it, may or may not have guessed it, I guess, we're gonna go back to our continuity equation. That's the other tool in our tool box.

We know that the volumetric discharge, letter Q, is equal to the cross-sectional area of Section 1 times the mean velocity of Section 1, V1, and that's also equal to the cross-sectional area of Section 3, A sub-3, times the velocity at Section 3, the mean velocity that is of Section 3, V with the subscript 3.

Now, if I rearrange everything in this equation, I can say that V1 is equal to Q divided by the cross-sectional area A1. Or simplifying that, I get Q over 45.5. Again, we knew that the depth was 6.5 and the cross-sectional width was 70 feet at that particular location of Section 1. By the same token, we can take Section 3 and we can say that the velocity at Section 3, V3, is equal to the volumetric discharge, Q, divided by the area at Section 3, A3. Simplifying that, we can say that that's equal to Q over 200. Again, we knew that the velocity or--excuse me--the depth was five feet and the cross-sectional width at that location, Section 3, was 40 feet. Again, that's 200 square feet for the cross-sectional area at Section 3.

Now, I'll go back to my equation that I had with the energy equation. Remember, we had V1 squared on the left-hand side. We had V3 squared on the right-hand side. We're going to plug these values in for V1 and V3 respectively. So instead of putting V1, I'm gonna say Q over 45.5. Again, remember, that was V1 squared. So now I have to have Q squared over 45.5 squared--I'm sorry--455 squared. I'll plug that in on the left-hand side of the equation. On the right-hand side, I'll do the same thing. Instead of putting V3, I put Q over A3 squared. Okay? Now, I'll plug that in, I've got Q squared. It's the only term that's unknown, and I can solve that fairly simply to determine that the discharge is 2000 cubic feet per second.

That is essentially our example problem. I'm hoping now that you will go in to Lesson 6 and work the problems in the textbook.