Example Problem Momentum Principle

SPEAKER: For our example problem for the momentum principle, we'd like to use the momentum principle to calculate the upstream depth that results from the drag force on a pier. The knowns that we have are that we are working with a 2-foot wide pier and the flume that it is in is 8 feet wide. The discharge, Q, is equal to 240 cubic feet per second. The downstream depth is measured, D2, at 5 feet and this is the normal depth as we will explain in a future lecture.

We have experimentally determined the drag force, F sub-d, to be equal to the quantity 2.1 times AF, which is the frontal area of the pier, times the velocity in the upstream channel, V1 squared. And they tell us in the example problem that we can ignore the bed shear and the weight of the water in the flume. Well, many times, the most difficult part of solving these sorts of problems is establishing where the control volume is. In this case, we're going to draw the control volume, a, b, c, d, such that the line a-b is along and just above the bed of the flume, line b-c is just far enough downstream of the pier to be out of the pier disturbance. Line c-d is above the entire water surface and pier and line d-a is just far enough upstream to be out of the pier disturbance. Using that control volume, the various forces that are acting on that, along line a-b, there's the water--excuse me, the water pressure force, F sub-p, that acts upward but this is balancing the weight force which is acting downward and they told us in the example problem to ignore this. There's also the bed shear force, F sub-tau which is acting upstream along the bottom. However, we can ignore this one as well based on the problem statement. Along line b-c, there's the hydrostatic force on the downstream end, F sub-2. And along d-a, there's the hydrostatic force on the upstream end, F sub-1. There are no forces acting along c-d.

Internally, there is the drag force of the pier, F sub-d is equal to 2.1 A sub-f V1 squared. I wanna note though that although we ignored the internal shear and pressure forces, the drag force is a principal cause of acceleration in this control volume so we need to consider that force. So the three forces that are remaining then are the drag force, F sub-d, and the two hydrostatic forces, F sub-1 and F sub-2. And we can determine them based on the equations that we've learned about already. In the problem statement, they tell us that F sub-d is equal to 2.1 times the frontal area,

A sub-f, times the velocity in the upstream channel, V sub-1 squared. And putting in the values that we know, the area is two times the depth upstream and so we get a solution for F sub-d equal to 4.2 times the depth in the upstream times the velocity in the upstream squared. Hydrostatic forces from Lesson 2 we know are equal to gamma times the width times the depth over 2. So for F sub-1, we calculate 62.4 for gamma and 8 feet wide for the flume. The depth is unknown so we leave that as D1 squared all over two which gives us a result of 249.6 D1 squared. F2 is 62.4 for gamma, 8 for the width of the flume, 5 for the depth and all of that over 2, giving us a result of 6240 pounds.

We need to now determine the velocities to be able to use the momentum equation and we use the continuity equation to determine these. Q is equal to VA or the velocity is equal to discharge over area. So V1 is 240 cfs over the width of the flume, 8, times the depth upstream, D1, giving us a value of 30 over D1. V2 in the downstream portion of the channel is 240 cubic feet per second divided by the area, 8 times 5 or an answer of 6.0 feet per second. We can now apply the momentum equation, the sum of the forces is equal to rho g times V2 minus V1. We can now apply the momentum equation, the sum of the forces equal to rho times Q times the quantity of V2 minus V1. The forces are F1 minus Fd minus F2 and the reason for that is that I am considering all of the forces to the right in the direction of flow to be positive and those that are against the direction of flow will be negative.

Substituting the various values we've already determined, we can then write the equation in terms of one or two variables and in this case, there will be two, 249.6 D1 squared minus 4.2 V1 squared minus 6240, those are our three forces. Rho is 1.49. Q is 240. V1 is six--excuse me, V2 is six. V1 is 30 over D1. At that point, we have two variables, D1 and V1. Well, let's replace V1 with what we determined using the continuity equation. In this case, we replace that V1 squared with 30 over D1 and we square that quantity. This gives us one equation with D1 as the only unknown. And simplifying them, you get the equation that's at the bottom of the screen, 2.49 times D1 squared plus 10,188 over D1 is equal to 9033.

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Now, this final equation can't be solved directly, it has to be solved using trial and error and we start with D1 equal to D2 which was 5 feet. Then looking at the table there, to the lower left of the screen, if D1 is equal to 5 then the left hand side of the equation, 249.6 D1 squared plus 10,188 over D1 is equal to 8277.6, that’s not equal to 9033 so let’s try again. Five point two gives us 8708.4. Five point three gives us 8933.5. Five point four gives us 9165 which is too high so we drop it back down. Five point thirty-five gives us 9048.5 which is still a little bit too high. Five point three-four gives us 9025.4 which is approximately equal to 9033. We could take the depth term out to further and further significant figures but they begin to lose their significance. Therefore, the depth in the upstream channel is calculated as 5.34 feet. This is the end of the example problem for Lesson 10. In the next section, we’ll use the flume to demonstrate several scenarios where the momentum principle is needed.