Example Problem: Forces on Submerged Objects

SPEAKER Charles Berenbrock: This is the second example for Lesson 2. In the first example, it was a simple pressure diagram that we solved for. In this example, it's a more complex pressure diagram that we'll be solving. This sample--this example is very similar to the first one that the stream depth is three feet. However, we put a head gate in this stream, at the bottom of the stream, the last two feet in the stream is a head gate. And we want to find the force on that head gate. Also we want to locate the resultant force on that head gate. Also the head gate--this head gate is four feet in width.

We use the same concepts as before. As you see here, this is a pressure diagram. And it's in rectangular form here, as you can see. However, it might be kind of tough to solve because we're only interested on the pressure on the gate, the last two or bottom feet on the gate. Well, to simplify this, what we want to do is break up the pressure diagram into two parts. The first part is a rectangular region, and the second part is a triangular region. We'll work with the rectangular region for the first part here. And as you see in this diagram, the pressure is constant throughout the depth. And this pressure is equal to a depth of one foot. So if you look at the equation there, it says the pressure at one foot is equal to the specific weight of water times the depth of one foot. And if you go through the rest of that calculation, the pressure there at one foot depth, it's 62.4 pounds per foot squared. This is also the average pressure for the whole rectangular region because it's the same throughout this rectangular region. The centroid part, or I should say--let's go back to the force part. The force then is the average pressure times the area. So if we take the average pressure of 62 pounds per foot squared multiply that by the area of the gate, which is two feet, and the width is four feet, the resultant force on the rectangular region is 499.2 pounds. The location of the force is half--or is midway through the gate. So I'm going to define it as one foot from the top of the gate, or it could be one foot from the bottom of the bed.

Now, let's go into solving the pressure diagram for the triangular region here. As you see here, the pressure is variable. The pressure at one foot, again, is the same as before. It's 62.4 pounds per foot squared. And then the--then we need to know what the pressure at the three-foot depth.

And it is the specific weight of water times a depth of three feet. That pressure then becomes 187.2 pounds per square foot. What we need to do now is determine what the average pressure for that area, for that triangular area. So if you're looking at the equation at the bottom of your screen there, you'll see that the pressure for the triangular is equal to the pressure at one foot plus the pressure at three feet divided by two. Doing that calculation, the math comes out to be 124.8 pounds per foot squared. Now, we need to determine what the force is from that--resultant from that triangular area. So, again, force is still equal to average pressure times the area. So average pressure again was 124.8 pounds per foot squared then we need to multiply it by the area, and the area is two feet in height and four feet in width. When we do that calculation, the resultant force on the triangular area is 998.4 pounds. As you recall before, the location of a force for a triangular area is two-thirds the length from the top of the triangle. And we're going to say this is the top of the gate and that becomes 1.33 feet from the top of the gate, where that resultant force occurs. However, we need to find the total resultant force for the entire pressure diagram or prism, and it is the resultant force, or the total force here, is the force from the rectangular region plus the force from the triangular region. So we have to sum both of these forces, and once we do that, we get the resultant force as 1497.6 pounds.

But we need to know also where does this resultant force occur? Well, we're using a--on the screen there, you'll see your next equation there. And the resultant force is equal to the rectangular region times a, say, a moment arm of that rectangular region plus the force of that triangular region times the moment arm of the triangular region, and then you'll divide it by the resultant force, and that will be your moment arm or the length, where the resultant force will occur. And in this case, the resultant force, if you go through all the math and you can see it down there, is 1.22 feet from the top of the gate. This was a little more complex, a diagram we had to solve for two pressure diagrams. One was triangular, and the other one was rectangular. When you get complex objects, you might have to do something like this. It's much easier to do this than, say, doing some integration work. This concludes this section.