Example Problem Dimension Analysis

SPEAKER Bob Holmes: Okay, it's time now to work a problem using our concepts that we've learned in dimensional analysis. And our challenge here in this problem is to derive an expression for the drag force on a smooth object as water moves past the object. Now, through the examination of the physics of this phenomenon, we can hypothesize that the drag force depends on the speed of the water, the size of the object, and the density and viscosity of the water. For size of the object, we're going to use a diameter term D.

Now, what I'm going to do, as we have listed in our previous steps, where I gave you sort of an outline of a process to go through, we first list that the drag force, F sub-D, is equal to as a function of the velocity of the flow times the diameter of the object times the density of the water, rho, times the dynamic viscosity of the fluid, mu. Now, don't get hung up here. This F sub-1 is just a convention that we use to denote that this is a function. So I might use F sub-2, which I will later in the problem, to denote that it's a slightly different function. Again, this is not an algebraic equation. This is the dimensional analysis. Our next step would be to list the dimensions on each one of these variables. The convention that we've used earlier, when I gave you the lecture, was to say that every time that you see brackets in this form where you have the variable by itself in bracketed terms, that means that on the other side of the equal sign, those are the dimensions that that particular variable contains. For this particular side on your screen, you'll see that the drag force, F sub-D, has dimensions of force. Likewise, the bracketed D, capital D, which is the diameter of the object, has dimensions of length. The velocity, in brackets, has dimensions of length over time. The density of the fluid, in this case, water, has dimensions of force times squared divided by the length dimension to the fourth power. Lastly, our dynamic viscosity has dimensions of force, time, the length squared.

Now, if I look at this, our next step is to look at how many basic dimensions that we have. Again, we have five variables here, but we have only three basic dimensions that are common to all, or not common to all, but at least are occurring at least once in each one of the variables. That is force, time and length.

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Now, our next step would be to apply the Buckingham pi theorem. Remember, the Buckingham pi theorem is G, which is the number of independent dimensionless groups, also called pi terms, is equal to K, which is the number of independent variables, minus R, which is the minimum number of basic dimensions. That is G is equal to five minus three, three being our number minimum number of dimensions that we have in the problem, five being the number of variables, we're going to come out with an answer of two for G. Again, we're going to have two pi terms, or two independent dimensionless groups, that we should be solving for in this problem.

Now, our next step is to start removing the commonality--removing the dimensions because we want to collapse this problem into something that we have dimensionless terms. In this case, we've used Buckingham pi and we should be solving for--we should be working this problem looking for two dimensionless terms. Now, in our rules, in our hints, we said let's always eliminate the force dimension first. Now I've selected a density term on the right side of the--of this dimensional analysis problem. And I--because it has the units of force in it, and it's force to the first power, I'm going to use that to divide any of the variables that have force terms. Now, remember, as I mentioned before, this is not--we don't use rules of algebra here. We do not have to divide everything on the right side of the equation by this rho term if it does not have a force term in it. For example, we're going to divide the dynamic viscosity term by the rho because both of them have force in it. And as the same token on the left-hand side of the equation, I'm going to divide the drag force, F sub-D, by the same rho term because it also has units of force, all right? So our next way to write this would be that F sub-D, the drag force, divided by the density is equal to the second function of velocity, diameter of the object and the--or the dynamic viscosity divided by density.

Now, I'm going to look at the dimensions left on the two variables that had new combinations we've made. The drag force divided by the density of the fluid as dimensions of length to the fourth power divided by the time to the second power. The other variable that we've altered is the dynamic viscosity when we divided it by the density of the fluid. That has dimensions of length squared per time.

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All right. I'm going to again--according to our steps that we've outlined in the previous lecture, I'm going to repeat step 4 again. This time, I'm going to try to eliminate the time variable. And again, as we said in those hints, we're going to use velocity to do that. So I'm going to divide all the variables that have time in them. You'll notice here that I've re-written this as the force, the drag force divided by rho, which we had done in the previous step, this time, because we have the units of time in there squared and that in the other combination, we're going to divide by the velocity squared in order to get rid of all the time units.

On the other side of the equation, we've--of this dimensional analysis, we're going to divide the old term of mu over rho. We're going to divide that by velocity to the first power because we only had the time unit in there to the first power that was left. So now we have a new function statement in the dimensional analysis that says the drag force, F sub-D, is divided by rho times V square. That's a function of the diameter and the dynamic viscosity divided by rho V. Again, because the diameter term D did not have any units of time in it, or any dimensions of time, we do not have to divide it by velocity.

Again, we look at our altered variables and see what the dimensions that are left. We have the dimensions of F sub-D over rho V squared. They have dimensions of L squared, length squared. The other variable that has been altered is the mu, the dynamic viscosity, divided by rho, which is the density of the fluid, times the velocity that has units of length.

Our last step is to eliminate these units of length, and so we have the diameter of the object left, and so we just divide through all the terms that have diameter in them or length in them, and so you can see that our final dimensional analysis, functionality is going to be the drag force, F sub-D divided by rho, V squared, D squared is equal to or is--or has a function of mu divided by D rho V.

That concludes our example problem for dimensional analysis. Please proceed to the book and work the problem that you have left to solve dimensional analysis.