**Flow Resistance Equations (Lesson 7)**

**SPEAKER Bob Holmes:** Hi. We've been talking about resistance and shear stresses and bed shear stresses, and now we're going to move in to actual flow resistance equations. Now, through multiple observations throughout time in the flume and in the field, we've noted that the bed shear stress is proportional to the square of the mean velocity. So you'll see that in equation form whereby we have the shear stress proportional to the velocity squared. The proportionality sign almost looks like a figure eight turned on its side with an open end. That's a sign for proportionality. Now, we can get rid of the proportionality sign and we can replace that with an equal sign if we put the coefficient C. That just says, "Okay, we know that we've got a relationship between the bed shear stress tau sub-zero and the square of the velocity, but we don't know exactly what it is." So we put this value that's unknown. We put that as a coefficient C.

Now, if we go a little bit further and we say that that mean velocity V, if we take the square root of that and we divide the shear stress, tau sub-zero, divided by the density of the fluid rho, we can then replace that C with a value of C prime. We haven't done anything wrong here because we don't really know what the coefficient C is anyway. So all we're saying is we're going to wrap up all these unknown into the C prime, and because we've done some further manipulation and we've taken the square root of both sides, we know that C is not going to be equal to C prime, right? So we just essentially put the C prime in its place.

Now, if you'll note on the right side of the equation, tau zero divided by rho, the square root of that quantity. That's simply the shear velocity. Again, we abbreviate shear velocity with U star. And that was derived in the last section, and we also discussed it in Lesson 5. Now, if we put that--what the actual definition of the shear velocity is instead of the U star, I plug that in the value of the square root of G R S sub-f, which is the acceleration of gravity, G, times the hydraulic radius, R, times the value of the friction slope, S sub-f, under the radical sign.

Antoine Chezy in 1769 conducted a number of experiments on both earthen canals in the Seine River. He concluded essentially the equation that you have on your screen, which is the mean velocity is equal to a coefficient C. Now, essentially that's just the coefficient of the roughness times the square root of R times the slope of the bed, S sub-O. Now, if we take a close look at
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this equation, we see that this is not dimensionally correct. The velocity has units of length over time and the value of $R_s$ to the one-half power has units of length to the one-half power, therefore in order to make this dimensionally correct, the value of $C$ needs some units attached.

Today, in many cases, you’ll see Chezy’s equation written as the mean velocity is equal to the value $C$, which is his roughness coefficient, divided by the square root of the acceleration of gravity, $G$, times the shear velocity, $U_{*}$.

Now, many efforts have been made to develop predictive equation to solve for $C$. One of the most famous was from an Irish engineer in the 1800s named Robert Manning. He came up with an equation for $C$ that was equal to $1.49$ over a Manning coefficient $N$, which is again a measure of the roughness, times the hydraulic radius, $R$, to the one-sixth power.

Now, if we go insert Manning’s work into the original Chezy equation, we come up with the famous Manning equation, which is used predominantly here in the United States as the mean velocity $V$ is equal to $1.49$ over $N$ times the hydraulic radius to the two-thirds power times the friction slope to the one half power. Now, if we combine that with the continuity equation, recall that’s the value of the volumetric discharge $Q$ is equal to cross-sectional area times the mean velocity $V$. If we combine those, we get $Q$ is equal to $1.49$ over $N$ times the cross-sectional area $A$ times the hydraulic radius to the two-thirds power times the friction slope to the one-half power. Dimensionally, the value of $N$, roughness coefficient, has the units of length to the one-sixth power, and the value for $1.49$ (the constant) has units of feet—or length to the one-half power divided by time. Again, that’s simply to make it dimensionally correct. But remember, this equation was derived experimentally (or empirically).

Now, if we look—the equation, as we have stated it, has been in English or British gravitational units. If we put that in SI units, the only difference between Manning’s equation for English system and Manning’s equation for SI, instead of having a constant of $1.49$, we have a value of our constant of one over Manning’s roughness coefficient. So that becomes $V$ is equal to one over Manning’s roughness $N$ times the hydraulic radius to the two-thirds power times the friction slope to the one-half power.
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Now, Manning and Chezy are the most common equations in the U.S. to describe flow and open channels with Manning being more popular. Now, I wanna bring out two cautionary notes on the assumptions of the Chezy and Manning equation. The first one is, is that the square of the velocity is proportional to the bed shear stress. That’s been empirically derived or through observation out in the field, in the laboratory. Also, the energy dissipation is caused entirely by boundary resistance. Remember, in the previous section, we talked a little bit about that where we said, “All right, our assumptions are that the energy dissipation or energy expended is totally due to the boundary resistance. The other case would be if we had some sort of form drag due to a bridge pier or some large boulders that are protruding up to the flow.”

That concludes this section of Lesson 7. I would now encourage you to proceed to the next section where we will cover an example problem.