Flume Demo Hydraulic Jump

SPEAKER Rick Huizinga: In this flume demonstration of the momentum principle, what we have created is a hydraulic jump. On the upstream side, we have a sluice gate which is up beyond our measurement--up beyond our scale. And what we have then is we've created a hydraulic jump--we have very high velocity flow in the upstream section, downstream it's much slower and deeper, and what we are going to do is we're going to measure the upstream water surface and the upstream velocity and then we're going to use the momentum principle to compute what the downstream depth is and then we'll compare it to what we actually have here in the flume.

So let's take a closer look at what we have at the upstream scale. We see there that the depth is approximately 13 millimeters from the bottom of the flume and you see the meniscus is varying quite a bit, the velocity fluctuates quite a bit when you have very thin flow like that but it's averaging at about 77 millimeters. So with those two numbers, a depth of 13 millimeters and a velocity head of 77 millimeters, let's now go to the computation section and compute some things using the momentum principle.

We're using the momentum principle to calculate the depth downstream from a hydraulic jump. Our knowns are that the flume is 100 millimeters wide, we're converted that to feet, 0.328 feet. We've volumetrically measured the discharge to be 0.057 cubic feet per second. We have the upstream depth, we measured it at 18 millimeters and convert that to feet, 0.059 feet. We also have the upstream velocity head which we measured at 77 millimeters but we need to subtract off the depth because, remember, the velocity head is the difference between the energy grade line and the water surface. So, 77 minus 18 millimeters leaves us with 59 millimeters. We convert that to feet, 0.194 feet.

Our control volume or our black box in the hydraulic jump situation extends from upstream of the jump to downstream of the jump. We look at the various forces that are acting on that control volume or black box and we see that we have the water pressure force acting upward, which balances the weight force acting downward and we're going to ignore that. We also have the bed shear force, F sub-tau, that's acting upstream and we're going to choose to ignore that one as well. We have the hydrostatic force on the downstream end, F sub-2, and we have the hydrostatic

force in the upstream end, F sub-1. Along the top, along c-d there's none and there are no internal forces in this particular case.

So let's determine the forces. Our hydrostatic forces from Lesson 2, F is equal to gamma times WD squared over 2. F1 we can compute directly, 62.4 for gamma, 0.328 for the width, 0.059 for the depth, we square that, and take all of that over 2 and we get a hydrostatic force on the upstream end of 0.036 pounds. Now, F2 we have to leave in terms of D2 because we have not determined the downstream depth yet. But we have gamma, we have the width of 0.328, we have D2 and we square it and take that all over 2, that leaves us with 10.23 times D2 squared. Now, we can determine the upstream velocity from our velocity head which we measured. Taking our upstream velocity head of 0.194 feet, we can compute V1 to be 3.53 feet per second.

Now, as a check, let's determine the upstream depth from the continuity equation, knowing that Q is equal to V times A, or the depth is equal to the discharge over the velocity times the width. Well,

we find out that when we plug in our Q of 0.057, our velocity of 3.53 and our width of 0.328, we get an upstream depth of 0.049, which doesn't check out with the 0.59 that we actually measured. Well, let's determine V1 and V2 using continuity then. V1, we can compute, is Q over the depth times the width. So 0.57 over 0.049 times 0.328 gives us a velocity in the upstream of 3.55 feet per second. This is very close to the V that we just computed. However, as you saw velocity head tends to fluctuate quite a bit and so this is a difficult thing to determine from these measurements. Let's use the V1 that we've computed here from continuity. V2 we'll compute is Q over A and we have to leave that one in terms of D2 so we have 0.057 for our discharge, 0.328 for our width which leaves us with 0.174 over D2.

Well, now we can use the momentum equation, the two forces are F1 and F2 and, of course, the right hand side of the equation is rho, Q times V2 minus V1. Let's substitute in the various values that we have determined. F1 we know is 0.036 pounds, F2 is 10.23 D2 squared, rho is 1.94, Q is 0.057, V2 is 0.174 over D2 and V1 is 3.55. Now, we have one equation with only one unknown,

which is D2, and so we can simplify that. But we wind up with an equation that looks like 10.23 times D2 squared plus 0.019 over D2 is equal to 0.357 and this is very difficult to compute directly

so we'll have to solve it using trial and error. So let's try a D2 of 0.2 feet and you see there as we work our way down, through the table in front of you, that if D2 is 0.2 feet, then the left hand side of the equation, 10.23 times D2 squared plus 0.019 over D2, gives us a value of 0.504 and that's not the same as 0.357 so that's not the right answer. Let's step it down. Let's take D2 of 0.18. You see that we get a value of 0.437, that's not quite it. If we take it down to where D2 is equal to 0.15, we find that the left hand side of the equation results in 0.357 and, therefore, D2 is equal to 0.15 feet. If we convert that to millimeters, we see that D2 is equal to 46. Now, let's go back to the flume and see what we actually measured for D2.

Well, now, that we've done the computations, let's take a look at what our downstream scale actually says. If we take a closer look at the downstream scale, we see that it reads about 52 millimeters. Now, this value, 52 millimeters, is slightly different than what we had calculated using our computation section but you can see that the momentum principle estimates this downstream depth fairly well. The reason for the difference is that we did not account for the friction effects along the bed of the flume but you can see that the momentum principle is a useful tool in determining the downstream and upstream depths of a hydraulic jump.