Free Body Diagram: Pressure and Depth

SPEAKER Bob Holmes: Okay. In this session, we're gonna talk about the relationship between the depth and the pressure of a fluid. But before we can actually get to that, we need to talk about the concept of a free-body diagram. Now a free body diagram is a cutaway view of the fluid or object in which the effect of any surface that is cut is replaced by the forces on that particular surface. You'll see on the illustration on the screen that I have a cutaway or a free-body diagram of an isolated block of fluid. On each of the three sides that have been cutaway, the water surface is at the top or the fluid surface is at the top, but the other three sides have been cutaway. And I've replaced each one of those with the pertinent forces that are applied to each of those surfaces. For example, on the left side on that vertical cutaway space, you'll see that we have a pressure, force, as it comes from left to right on that block. We also have a shear force or a shear stress along that particular vertical wall. On the very bottom, we have a pressure of the fluid pushing up against that particular cutaway side. We also have a shear stress or a shear force along that bottom. And then on the right side or the right vertical side of that cutaway, we have a shear stress and a pressure force.

Now if the fluid is at rest, our shear stress on every surface is going to be equal to zero because we have no rate of deformation. Remember that, it was from the previous concept of Newton's Law of Friction. We also wanna note on this particular cutaway that we have a width of the fluid free body of DX. We have a thickness into the screen or into the paper that you're looking at or the slide that you're looking at of one foot. So that will define our volumetric size. We also have a height equal to Y. If I wanna compute the pressure at the bottom of this particular free-body diagram, I'm going to have to, again, borrow from Newton's Second Law of Friction which says that the sum of the forces is equal to the mass times acceleration. Again, if the acceleration, which we've already said, we don't have any movement in this particular free body, the acceleration is equal to zero. So we say that the sum of the forces in the X direction are equal to zero, and the sum of the forces in the Y direction are equal to zero. Now because all the sides are vertical and all values of the shear stress are equal to zero, if I sum the forces in the Y direction,

the only force that I have going down is the weight force of that particular free body, and the force coming up is the pressure force which is the pressure, P, on that bottom surface times the cross-sectional area, which is DX, times one unit of width into the board. So, again, I would have the weight force which is negative going down. I have the pressure force which is positive pushing up against that free body, which is the value of the pressure times DX times one. Again, pressure has units of pounds per square foot. The cross-sectional area has units of square foot, that's the DX, distance wide, times the one foot deep into the board or into the slide to give me my area. Now, if I say that the weight is equal to P times DX times one, that only gives me so far. Remember I'm trying to get at a concept to be able to tell you or to derive the relationship between depth into the fluid and pressure. So now I need to solve for what the weight value is. Now I know that if I have the unit weight of the fluid, which is the Greek letter gamma, that's based on our previous sections, that's the weight of the fluid per unit volume. If I multiply that gamma times the actual volume of the fluid, I would get the weight because if I have weight per unit volume times the volume, the volumes cancel out and I get weight as my remaining dimension or unit. So I'm going to say that the weight of that particular fluid is gamma times the depth Y times DX, which is the width of the free-body diagram, times one foot in. If I put that into the equation for the weight and I do some algebraic manipulation, again the DX will cancel on each side, the one cancels on each side, I get that the pressure is equal to gamma times the depth Y. This demonstrates that the pressure increases linearly with depth below the surface. Okay. This is an important concept because in any kind of stream gauging application that we have, a lot of our sensors out there on the streams and rivers that we gauge, you sense pressure. But we don't wanna know what the pressure is. We wanna know what the depth or the water surface elevation is. So that's the concept that we have. We take advantage of this concept, pressure is equal to gamma times Y. As long as we know the pressure, we can back out a value of the depth over the particular sensor that we have in the water. That concludes this section on pressure.