Hydraulics of Sharp Crested Weirs

SPEAKER Charles Berenbrock: Now we’re going to look at the hydraulics of a sharp-crested weir. Note in this diagram where Section 1 is, it’s upstream of the sharp-crested weir—it’s upstream at a sufficient distance to where the sharp-crested weir is not affecting any of the flow at the section. Again, the height of the weir is denoted by “P”, $H_1$ represents the water surface plus the velocity head at Section 1. Again, at a sharp-crested weir, it is forced to go through critical depth. So right at the point at 3, critical depth occurs and that’s denoted by $D_c$ again.

There is no Cross-section 2 in this. It does not exist. You only have Cross-section 1 and Cross-section 3. For sharp-crested weirs, the weir face is very sharp. The weir length, $L$, is very small. Thus, your contraction ratio $h/L$ is greater than 1. Also for sharp-crested weirs, the nappe will spring clear of the weir. And as you see in this photo of a weir, the weir length is small, the weir face is sharp, and we see a nappe is well ventilated below.

Section 3 is usually immediately downstream of the weir plate right there. And because the nappe is not fully supported from below, the pressure on the bottom of the nappe is zero. Therefore, the hydraulic head at cross-section 3 is not the critical depth but is an average potential energy of water passing section 3. And for a rectangular-shaped weir, the average potential energy is the critical depth divided by 2. Basically, that’s just the centroid of the rectangular area that represents the rectangular weir. Now if we had a triangular-shaped weir, the potential energy would be two-thirds times the critical depth.

Now let’s write the energy equation between cross-sections 1 and 3 for the sharp-crested weir, and we’re assuming that it’s a rectangular-shaped weir. And when you do that, you’d see the following equations. It’s very similar to the broad-crested weir. However, note that the depth is replaced by the potential energy at Cross-section 3 and it’s represented by the critical depth divided by 2.

Because the upstream depth, $h_1$, is large compared to the depth of the weir, critical depth, we can assume that our velocity is equal to zero. Also, we assume that our friction loss between 1 and 3 is also zero because our friction head losses—I should say our friction is very small, too. Because of those things, then we can rearrange the energy equation and the resulting that takes place is $v$, 

$\Delta v = \frac{1}{2} \frac{H_1}{D_c}$ 

$\Delta h = \frac{1}{2} \frac{H_1}{D_c}$ 

$\Delta x = \frac{1}{2} \frac{H_1}{D_c}$ 

$\Delta y = \frac{1}{2} \frac{H_1}{D_c}$ 

$\Delta z = \frac{1}{2} \frac{H_1}{D_c}$
critical velocity \( (v_c) \), is equal to the square root of 4 times \( g \) divided by 3 times the quantity of 1 plus \( k_e \) multiplied by the square root of \( H_1 \).

Then to attain discharge, we just multiply both sides of the equation by area or by width times depth. And when we do that, you see the resulting equation. However, we’re going to simplify this equation by having the contraction ratio where “C” is equal to two-thirds times the square root of 4 times \( g \) divided by 3 times the quantity of 1 plus \( k_e \). We insert that into the energy equation and the result in the equation simplifies to \( Q \) is equal to \( C \) times \( b \) times \( H \) to the three halves power.

Now, remember that \( b \) is the width of our weir. Theoretical values for \( C \) for a sharp-crested weir and for the rectangular plate, I should say, weir, ranges from 4.37 to 3.57 as \( k_e \) ranges from 0 to 0.5.

Hulsing noted that he obtained a minimum \( C \) value of 3.27 for a very small weir. He also noted that he obtained a maximum \( C \) value of 4.29. Hulsing gives a very complete description of weirs and it is recommended that if you need to more information to go to that reference. Also, if you need to use a correction coefficient for this rectangular weir, you can and it’s still again denoted by \( C \) equals \( C' \) times \( k_c \) times \( k_f \) or other losses in the system. Also, you’ll need to correct--have a correction if the nappe is not fully ventilated.

Now, the following diagram shows a triangular weir or a V-notch weir. This type of weir permits a very accurate measurement at low discharges, and it does better than a rectangular weir but remember again that this is for very low discharges. Notice in this diagram where the centroid of discharge occurs in the weir. Also note where a theta (\( \theta \)) is. \( \theta \) is the angle of the entire weir there or the V-notch.

Now let’s write the energy equation between cross-sections 1 and 3. Again, it’s very similar to the rectangular weir equation except that we replace the depth as two-thirds of critical depth. Note that the critical depth \( D_c \) is really 4 divided by 5 \( H_1 \). The critical velocity then becomes the square root of 14 times \( g \) divided by 15 times the quantity of 1 plus \( k_e \) times \( H_1 \) to the one-half power (\( H_1^{1.5} \)).
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Now if we substitute these in to the energy equation, we obtain the following equation. Let’s simply this equation by using the correction coefficient $C$. It now becomes $16$ divided by $25$ times the square root of $14$ times $g$ divided by $15$ times the quantity of $1$ plus $k_w$.

Putting that back into the energy equation, the energy equation becomes $Q$ equals tangent times the quantity of theta divided by $2$ times $C$ times $H$ to the three halves power. Thus, we’ll use this equation for the V-notch weir. When $k_w$ equals 0.5, $C$ equals 2.86. And Hulsing also noted that $C$ ranges from 2.46 for a 60-degree weir, that means theta ($\theta$) equals 60 degrees, to 2.48 for a 90-degree weir. Also, you’ll need to correct if submergence occurs and this is--the following equation shows that where you use $k_t$ and there--and $k_t$ means the tailwater elevation above the weir crest.

There are other sharp-crested weirs and they are used for other special purpose. In other words, the following diagram shows a trapezoidal weir. And then the following slide also shows four other weirs. For example, the Sutro or proportional weir, an approximate linear weir, approximate exponential weir, and the Poebing weir. You can obtain the final equations for them, too, if you--you should use the energy equation, the basic equation, and then you have to find out where the critical depths occur for each of these weirs and then substitute those in to find what $Q$ equals.

This is the end of the lecture portion of Lesson 18. In the next section, we will use the flume to demonstrate these concepts.