Key Concepts and Methodology

SPEAKER Rick Huizinga: This is Lesson 15: Water Surface Profile Computations. In Lesson 13, we presented the conceptual idea of water surface profiles. But in this lesson, we're going to actually show you how to compute them. We're going to use several key concepts, and I'd like to cover those briefly before we start the lesson so that you know where we're going. The step backwater method that I'll be presenting uses the energy equation and I'm going to use the subscripts u and d to designate upstream and downstream. And so you see the energy equation there on your screen, alpha-u, V-u squared over 2g plus h-u, and in this case, the h combines both the z and the y terms that we've been using so far. And that's all equal to alpha d times V-d squared over 2g plus h-d plus head losses between the upstream and downstream sections.

For sub-critical flow, we work from downstream to upstream. So, we can rewrite the energy equation such that we're solving for the upstream depth. H-u is equal to h-d plus alpha–d V-d squared over 2g minus alpha–u V-u squared over 2g plus h-f plus any eddy losses, h sub-e. The head loss due to friction, we compute as the average of the friction loss, s sub-f bar times L, where s sub-f bar is the square root of the product of the friction loss in the upstream and downstream cross-section and this is called the geometric mean. And finally, the eddy losses, h sub-e, are computed as a coefficient k sub-e times the absolute value of the difference in the velocity heads.

Now, water surface profiles are very common open-channel hydraulics problem. We need to determine the water surface profile of the stream under specific discharge conditions and channel conditions. This is also called the backwater curve although this term is specifically applies to water surface profiles in a subcritical flow condition. We used the water surface profiles to determine parameters for flood control works or highways or bridges and, of course, these sort of structures are used to divert, withstand or convey floods. We'll use them to determine tailwater ratings for hydroelectric dams, canal head works, and energy dissipators or for flood prone areas for emergency management issues and insurance or building permitting.

Now back in Lesson 13, I showed the conceptual idea of what a water surface profile looks like.

But now we're going to actually compute those water surface profiles. And to do that, we solve the energy equation for gradually varied flow. There are three methods to compute water surface

profiles. There's one called the graphical integration method, there is a direct integration method, and there's also a step method and this is the one we'll be using to describe water surface profile computations. And I must say at this point that I thank God very much for computers because to do this by hand is a tedious thing but computers, of course, handle that sort of tedium very well.

In the step method, we divide the channel into short reaches which have a small change in channel geometry. This permits the method to be used for all types of channels whether they're uniform or non-uniform artificial, and even if they're non-prismatic natural channels. And by limiting the reach length such that there are small changes in conveyance, we create a gradually varied flow situation. The first thing we must do is determine the flow regime and we learned how to do this in Lesson 11. If it's subcritical flow, then the control is at the downstream end and we compute our computations in an upstream direction. If it's a supercritical flow regime, the control is at the upstream end and we proceed downstream. We then determine which flow profile to expect, as I showed you in Lesson 13, whether it's an M1 or S3. And this is helpful to predict the shape of the curve and to know how to choose the next depth point in the problem. Well, what you have in front of you right now is a graph that shows basically an upstream and downstream end of a reach. You see that the flow is moving from left to right and this is basically the definition sketch of what we're working with.

We have our datum at the bottom and everything is measured from that datum. In the upstream reach, you have h sub-u which goes from the datum to the water surface, and then there's the velocity head in the upstream reach, alpha–u V-u squared over 2g. And the same is true on the downstream end, you have the depth of water from the datum, h sub-d, and you have alpha–d V-d squared over 2g. And then, of course, there's the head loss term at the top. Well, we determine the water surface elevation at the end of each reach starting at the point of control by solving the energy equation between the two ends of the reach. And we use the energy equation alpha–u V-u

squared over 2g plus h-u is equal to alpha–d V-d squared over 2g plus h-d plus any head losses, h sub-L. And again, h is equal to the depth plus the potential head z.

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Let's talk about head losses for a moment. The total head loss in the reach, h sub-L, is equal to the friction losses and any eddy losses. The head loss due to friction, we compute as the average friction loss, s sub-f bar, times the length of the reach L. The eddy losses, we represent with the coefficient k sub-e times the absolute value of the difference in the velocity heads from upstream to downstream. Now, these eddy losses are due to expansion and contraction and we've introduce that concept in Lesson 14. We used this coefficient k sub-e to relate the losses due to the change in velocity between the upstream and downstream ends of the reach, and we used the absolute value of that so that it's always a positive loss. This allows us to model 3D flow using 1D theory. In non-prismatic channels, eddy losses can be substantial, but for short reaches, we often assume that these eddy losses are equal to zero.

So we can rewrite the energy equation for subcritical flow which we compute from downstream to upstream. And so we can rearrange the energy equation and you see that at the bottom of your screen. The upstream depth, h sub-u, is equal to the downstream depth, h sub-d, plus alpha–d V-d squared over 2g minus alpha-u V-u squared over 2g plus the friction losses, h sub-f, plus the eddy losses h sub-e.

Now, let's take a look again at the friction losses. Typically, the friction slope is gonna vary along the reach. The actual friction loss would be determined by integrating the area under the curve that you see there. If our reach is represented by that squiggly line at the bottom of the graph, you could see that the friction loss may actually vary as you move along the reach. We tend to average it over the entire area and we would integrate the area under that curve using calculus. And you see that there the friction loss would be equal to the integral from the downstream section to the upstream section of s sub-f times dx. However, we find that the average s sub-f bar times the length approximates the area into that curve very well.

Now, there are several methods to determine s sub-f bar. There's the weighted average method which says that s sub-f bar is equal to one times the Greek symbol nu--I think that's correct anyway--times the friction slope at the downstream end plus nu times the friction slope at the upstream end, where nu is equal to one-half for an arithmetic average or nu is equal to pi over four for an elliptic average.

One of the other methods is the geometric mean and this is preferred by the USGS, and it says that the friction slope, s sub-f bar, is equal to the square root of the product of the friction slope at the downstream end and the upstream end. There are the two others, one that's called the harmonic mean which says that the average friction slope, s sub-f bar, is equal to two times the friction slope at the downstream end times the friction slope at the upstream end over the friction slope at the downstream end plus the friction slope at the upstream end. Or you can use the average conveyance method which says that the s sub-f bar is equal to the discharge in the upstream end plus the discharge in the downstream end over the conveyance in the upstream end plus the conveyance in the downstream end all taken to this--raised to the power of two. Now, all of these methods will yield satisfactory results and you can see Table 15-1 in your text for some guidance. But the USGS prefers the geometric mean and that's what we will use in our computations.

This is the end of the lecture portion for Lesson 15. In the next section, I'll work through an example problem that will help reinforce the concept of this lesson.