Rapidly Varied Flow at Constrictions

SPEAKER Charles Berenbrock: Welcome. This is lesson 16 on rapidly varied flow at constrictions. This lesson describes the theory or the idea behind using a constriction as a flowmeter. The energy equation is used to relate the change in the head to the discharge. In order to use this method, the upstream flow must be subcritical. The water surface elevation, or you can use high-water marks, for example, must be known and also the channel geometry of each cross section must be known. Now, examples of constrictions are like bridge abutments and maybe bedrock narrowing of the channel.

In this slide, it shows the Pearl River near the I-10 freeway in Mississippi. The bridge is a constriction. You can see the abutments coming out into the floodplain and near the river is where the bridge is actually located. Also, in this bridge constriction example on Long Creek near Quitman, Mississippi, you can see the actual embankments from the bridge going out into this creek here. In this slide, you'll see this is a bedrock narrowing example. It's on the Big Lost River at the Idaho National Engineering Laboratory in Idaho. In the upper left hand corner, you can see that the bedrock there, which is all volcanic, has narrowed the channel, and then the lower slide shows water in the channel. This slide is an example of the contours of the elevation of the bedrock narrowing. And you can see at site two, which was the previous photos, that it is quite narrow at that site. In fact, it narrows down to 20 feet across whereas the natural channel is usually a hundred feet across. These are just some examples of bedrock narrowing. We're going to be looking, in this lesson, at solving for the upstream water surface elevation using the energy equation. We're also going to solve for discharge. And then we're going to use Matthia's procedure, which is a practical application where he refined solving for discharge.

Now, let's look for solving the upstream water surface elevation. In this example, you'll see a constriction here. The natural channel is 500 feet across whereas at the constriction it's a hundred feet across. We're going to be placing some sections here, and you can see that section 1 is upstream of the constriction and it's in where the width of the channel is 500 feet. Section 2 and 2A are right near the entrance to the constriction and then section 3 and 3A are near the exit of that constriction. And then section 4 is, well, it's really off the screen. It's way downstream. This is just a theoretical example of a constriction that we're going to be walking through here.

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The first thing [Step 1] you should do is compute the normal depth and the critical depth. Normal depth is y sub-n (y_n) and critical depth is y sub-c (y_c) for both the contracted section and the normal section, the wide channel there. So you compute the normal depths by using Manning's n equation. Now, you might have to do a trial and error approach to do this. And then you can compute the critical depths, y sub-c (y_c), by that equation where D sub-c (D_c), your depth critical, is equal to the Q squared divided by the g, all that to the power of 1/3. Now, this equation is true for a rectangular cross section. It would not be true if your cross section, let's say, was trapezoidal or rounded, sub-rounded. You can look to references like Chow to find what the critical equation for those types of channels would be. In the second step [Step 2] what you would do is determine the flow regime downstream and within the cross section. So you would determine, let's say, what the Froude number is. The Froude number would tell you what the flow regime is. A Froude number of less than 1 would say that the regime is subcritical. A Froude number greater than 1 would mean that the channel section is super critical. After you've determine that, then you compute your velocities at 3A [Step 3]. Now, 3A is just downstream of the exit and then also you compute the velocity head, that's v sub-3A (v_{3A})squared divided by 2g for the exit section there. The next step [Step 4] is that you would write the energy equation across the outlet, in other words between section 3A and 3. And if you do that, you'll see the resulting equation there. You'll see that alpha sub-3 times (α_3) v sub-3 (v_3) squared divided by 2g. Now that's the energy head or, I mean, the velocity head at section 3. Remember that alpha sub-3 (α_3) is your Coriolis coefficient. Your velocity head plus your depth at section 3 plus your datum at section 3 is equal to the velocity head at 3A plus your depth at 3A plus your datum at 3A and then some exit loss. And that's represented by k sub-e (k_e), multiply it by the quantity of alpha-3, v3 squared divided by 2g minus alpha-3A velocity at 3A squared, all that divided by 2g, that is, again, your exit loss. And I kind of defined them here. You can see the red arrows with the definitions of each one below that. Remember that k sub-e (k_e) is the coefficient for energy losses due to the expansion of the flow. And you can see where the "quantity" is, that's the expansion losses between 3 and 3A. Now, if we assume that there is no friction term losses, because L, the length between 3 and 3A--3A is very narrow, it's very small--so we can assume that your friction

loss is 0. And also if we assume that your coefficient for energy losses due to the expansion is 1 and then the alphas are equal, we'll note that the equation will reduce to where D3 (D_3) is equal to D3A (D_{3A}). In other words, that the water surface is continuous at that expansion. However, if there are some expansion losses in that flow, you have to keep that term in. The next step [Step 6] is to determine a specific energy at 3, and you can do that by the next equation where E is a specific energy and it equals "h" sub-3 (h_3) plus your velocity head at 3 plus your exit loss. The next step [Step 7] is to determine the flow profile in the contracted reach. For example, if the flow is between critical and normal, you'll assume an M2 profile. And the resulting graph will show you what the M2 profile looks like from section 3 to section 2A. To obtain that profile, however, you use a step-backwater methods that you determined from a previous lesson to compute the depth at 2A [Step 8]. Once you determine the depth at 2A, then you can compute the velocity at 2A, the velocity head at 2A, and then also the energy, specific energy at 2A [Step 9]. However, you need to also compute the head losses between 2A and 3. And then to compute the depth at 2 [Step 10], you use the energy equation between 2 and 2A to do that. And then after that you can compute the velocity at 2, the velocity head at 2 and the energy losses at 2 to determine the specific energy at 2 [Step 11]. Now, remember that the contraction, the energy losses due to the contraction of the flow, "h" sub-e2 (h_{e2}). You ought to determine that.

The next step, as you notice, is 12 [Step 12], is you determine the flow profile between D_2 and D_1. For example, if y_1 is greater than your normal depth, which is y sub-n1, you'll assume an M1 curve. If y_1 is between, let's say, your critical depth and your normal depth, you would use an M2 curve. To obtain this curve, you use the step water procedures from the previous lessons to compute D_1. Once D_1 is computed, then you can obtain the velocity at section 1, the velocity head at section 1, also the velocity, I mean, the friction losses between 1 and 2 to obtain what your specific energy is at 1. If you look to this graph, you'll see the M1 curve shown on here. And for this one, we said y_1 is greater than your normal depth of y_1 and so that's why you would have an M1 curve.