Similitude

SPEAKER Bob Holmes: Okay. As we mentioned in the introduction, at times it's desirable to do physical modeling where we actually construct a scale model of the actual prototype and determine exactly what the behavior is gonna be prior to building some expensive hydraulic structure. Now the similitude concept is used to ensure measurements are taken in models that can be interpreted for the prototype. This similitude concept or similarity between the model and prototype require that certain laws of similitude are satisfied.

Now we have three basic types of similarity. We have geometric similarity. We have kinematic similarity, and we have dynamic similarity. Geometric similarity is basically the ratio of corresponding lengths between the model and the prototype. For example, in your figure on your slide, you see that you have a width in the model, which is abbreviated capital B sub-m, and you have a width in the prototype, which is abbreviated capital B sub-p. At the upstream approach section, we need to make sure those scale with the width through the contracted section, which is abbreviated small letter "b" sub-m in the model and small letter "b" sub-p in the prototype. In addition, we have a width of the contraction or longitudinal length of the contraction. You can think of it as like a bridge or a road width. That would be abbreviated capital L sub-m, for the model, and capital L sub-p for the prototype. We would wanna make sure those ratios remain constant for all aspects of the model.

Now there are some corollaries that we wanna discuss here in terms of geometric similarity. The corresponding areas vary with the squares of their linear dimensions. In other words, if I have the ratio of the area of the model to the area of the prototype, that would be equal to the length of the model divided by the length of the prototype, that quantity squared.

Another corollary we have for geometric similarity is that the volumes vary with the cubes of their linear dimensions. The volume of the model divided by the volume of the prototype is equal to the length of the model divided by the length of the prototype, that quantity cubed.

Now kinematic similarity is where we have the fluid flow ratio of both the model and the prototype behaving with similar time rates of change. Thus, any corresponding velocity or acceleration are

the same throughout the flow. This would imply that the fluid streamlines are similar. We're gonna get into a discussion of fluid streamlines later on in Lesson 4.

Our last similarity we're gonna discuss in this lesson is dynamic similarity. Ratios of all the forces acting on corresponding fluid particles and boundary surfaces in the model and the prototype, those fluid forces must be constant. Now let's talk a little bit about the forces on fluid flow. We have basically six forces here. We have the gravity force, which is abbreviated capital F sub-g. We have the pressure force, which is abbreviated capital F sub-p. We have the viscosity force, which is abbreviated capital F sub-v. We have the elasticity force, which is abbreviated capital F sub-e. We have a surface tension force, which is abbreviated capital F sub-t. And we have the inertial force, which is abbreviated capital F sub-i.

Now we're gonna go through each one of those and talk a little bit about those. Each force is governed by the relations between the dynamic and kinematic properties of the flow and by the physical properties of the fluid. For example, for the viscous force, capital F sub-v, that's basically the shear stress multiplied by an area. Okay? If you remember from Lesson 1, we talked about Newton's Law of Friction. And we had a relationship between the shear stress and the deformation rate, which was DVDY. Now in your slide, you see that we abbreviate that delta V delta Y, and we multiply it times the dynamic viscosity, which is the Greek letter mu. Obviously we plugged that in and we still multiply that entire quantity by the cross-sectional area A, or not necessarily cross-sectional area, but the area that the force is applied across or the shear stresses that are applied across.

Now, if we remove those concepts from DVDY and the various kinds of constants or abbreviations there, if we just replace those with the dimensions, we can come up with something called characteristic velocity and characteristic length. And so we get a simplification of that by saying that the viscous force is equal to the mu value, which is the dynamic viscosity, times the characteristic velocity V times the characteristic length L. And we'll get a little more into detail on that in a second.

Now let's look at the rest of the forces here. We got the pressure force, which is pressure times

the characteristic length squared. We have the inertial force, which again that's inertia and that's due to the movement of the fluid. That would be the mass times acceleration. If we replaced, again, the mass and acceleration by their characteristic lengths and velocities, we can see that that is simplified to be the density of the fluid times the characteristic length squared times the characteristic velocity squared. If we look at the gravity force, again, we have the mass times the acceleration of gravity. Remember, that's different than the inertial force because the inertial force is the acceleration of the fluid itself. Whereas the gravity force is the--the acceleration term there is the acceleration of gravity as opposed to the acceleration of the fluid. There we have the gravity force F sub-g is related to or equal to the density of the fluid times the characteristic length cubed times the acceleration of gravity G.

You can move on down through these and see that the viscous force is equal to mu, which is the dynamic viscosity, times the characteristic velocity V times the characteristic length L. The elasticity force or compressibility is a function of the compressibility coefficient E times the characteristic length squared. This compressibility factor E, that's based on the fluid itself.

Our last force here we're gonna talk about is the surface tension force where we have the surface tension per unit length, which is abbreviated by the Greek letter sigma, times the characteristic length L.

Now if we look at Newton's Second Law, which says, "That the sum of the forces is equal to the mass times the acceleration." That is summarized on your graph right now, where we have F sub-g plus the F sub-p, pressure force, plus the viscous force, F sub-v, plus the elasticity force plus the surface tension force, F sub-t, equals F sub-i, or that is also known as the inertial force, which is equal to the mass times the acceleration of the fluid.

Now to fulfill kinematic similarity with the ratios and accelerations and velocity, I might add, are the same in the model and the prototype. The ratio of the inertial forces must also be the same. So you'll see the equation pop up on your screen now, which says that the inertial force in the model F sub-i, in parenths, to the subscript m divided by the inertial force of the prototype F sub-i, in parenths, with a subscript p is equal to the sum of the forces for the model divided by the sum

of the forces for the prototype. Now if I restate that, I can rearrange the terms and I can put the inertial forces in the prototype underneath the sum of the forces of pressure, gravity, viscosity, elasticity, and surface tension in the prototype and I balance that with the ratio of those same forces in the model. If I break those into components, you can see that I make the ratio of every one of those particular forces that we have on that fluid control volume Fp over Fi for the prototype. I have the force of the gravity, F sub-g, over the Fi for the inertial force for the prototype and I go through that I can see that I have that on both sides. We can break those into individual components. Now if I want to ensure that the ratio of the inertial forces must be the same between the model and the prototype, that is to satisfy kinematic similarity, we will be successful if the ratio of the inertial force to each component force is constant between the model and the prototype.

For example, if the acceleration is due to pressure, it must be similar in both the prototype and the model. Okay? So on your screen, you'll see that the inertial force, F sub-i, over the pressure force, F sub-p, in the model on the left-hand side of the equation has to be equal to the inertial force divided by the pressure force of the prototype on the right side. In addition, we can look at it under the viscous force so we can see that those accelerations due to the viscous force must be similar in both the model and the prototype, and that equation now appears. We go on and on throughout the rest of these for gravity, elasticity and surface tension and you get the idea.

Now, when I look at these ratios, this is where the various dimensionalist numbers that we use in hydraulics and fluid mechanics comes into play. If you look at the five force ratios, you can see that if I have the ratio of the inertial force to the pressure force, that is something we call the square of the Euler number. That's E-U-L-E-R. That's pronounced Euler. That is the density of the fluid times the characteristic velocity squared divided by the pressure. The next term down is the Reynolds number, which we talked about as being the ratio of the inertial forces to the viscous forces. That's the characteristic velocity times the density, rho, times the characteristic length divided by the dynamic viscosity. Now, this is going to be a number that's very crucial for us in fluid mechanics because we look at this number when we compute it, that gives us the

difference between turbulent and laminar flow. As we will discuss in Lesson 5, we have turbulent flow when we have the variation of velocity, if you were to put a spot of dye in the fluid, you would see it disperse rather quickly as there's a lot of turbulence or agitation in the fluid particles. If we had laminar flow, that implies--just like you have lamination in boards or material, if you have laminar flow that means the fluid particles are moving in a laminar fashion or moving parallel to each other, so you don't have a lot of interchange between the water particles. If you put a spot of dye in there, that dye would stay fairly concentrated as it moves through the flow. The Reynolds number is used as a point of demarcation between turbulent and laminar flow. And we will discuss that a little more in later lectures.

Now, if I look at the ratio of the inertial forces to the gravity forces, you can see that that is the square of the Froude number. The Froude number being--or the square of the Froude number being the characteristic velocity squared divided by the characteristic length times the acceleration of gravity G. This number is used to tell us the difference between critical and subcritical flow. For those of you in mountainous environments that are working on rivers and streams, you might encounter critical flow or super critical flow. And essentially, when the Froude number is equal to one, that is the demarcation. Less than one is in the range of subcritical flow and greater than one, for the Froude number, is the range of supercritical flow with the value near one being what we call a critical flow. And we will talk about that more when we get into broad crested weirs and some of our critical flow equations that we have whenever we have a method to determine what the flow is when we send it through critical flow. Partial flumes, for example, are what's called a critical flow flume where we actually forced a flow into a critical situation where we get a unique relationship between stage and discharge.

Now, the ratio between the inertial forces and the elasticity forces is the Mach number. This is a ratio that we're not gonna use a lot because in open channel hydraulics we don't really worry about elasticity of the fluid. Those of you that might have interest in aeronautics and things like that, you've heard of the Mach number. That's basically when you break the speed of sound. And essentially, when you get into those higher velocities such as a jet airplane flying through the air,

air being a fluid, that's where you get into having to worry about compressibility. And that's where we get the Mach number and when we talk about the importance of the Mach number. The last thing we're gonna talk about is the Weber number. And that's the ratio of the inertial forces to the surface tension forces. And we're not gonna concern ourselves too much about that either in this particular course because we don't have too many situations where surface tension becomes a problem. If you get into water quality modeling and some of the other aspects, Weber number when you look at bubble flumes or flumes in general, things like that, that might become more of an issue. Before general fluid mechanics, basic hydraulics course, we're not gonna concern ourselves too much with that.

Now, most of our engineering problems we are concerned about, our hydraulics problems that is, we disregard compressibility and surface tension like I said, which means that the Euler, Reynolds and Froude number are more of what we're concerned about. Now, if we require the Reynolds number and the Froude numbers to be equal between the model and the prototype, that automatically is going to make our Euler number equivalent, all right? Now, again, we're ignoring the Mach and the Weber number. However, we have a little bit of a problem. Complete dynamic similarity is usually impossible to attain when we are concerned trying to keep a Froude and a Reynolds number both constant between the model and the prototype.

Let me give you an example. Let's tell the example of the contained or contracted flow problem that you see in plan form on your screen. Let's look at that again. You have a velocity in the prototype of eight feet per second in the approached section. You have a width of the channel, D sub-p, 10 feet. Let's say that you wanna construct a completely dynamics and geometrics similar model of this prototype. Now, if we assume this complete similarity, we have to have the Reynolds number of the prototype equaling the Reynolds number in the model and we have to have the Froude number in the prototype equal the Froude number in the model.

What's gonna be our model depth and velocity? So let's take a look at that. At 70 degrees Fahrenheit, we can look up in our tables in our textbook that the dynamic viscosity is 2.04 times 10 to the minus-5 pound seconds per foot cube--or foot square, excuse me. We also will assume

that the density of the fluid is 1.94 slugs per cubic foot. Now, if we go through when calculate the Reynolds number of the prototype, we will throw up, and we're gonna use our characteristic velocity in this case as the value of our approach velocity and we use the characteristic length being our depth, 10 feet, we can calculate a Reynolds number for the prototype of 7.6 times 10 to the sixth, okay? And again, that's dimensionless, so there are no units on that. We'll also calculate the Froude number of the prototype, and that's essentially the characteristic velocity divided by the square root of the acceleration of gravity times the characteristic length. Again, we are using our characteristic velocity as the approach cross-section velocity of eight feet per second and we're gonna use the characteristic length equivalent to our depth. This is usually what you're going to have in most of your instances when you're working an open channel flow problem. It will be something--in this case, we're interested in the approach velocity, so that becomes our characteristic velocity, and depth is a typical characteristic length that we're gonna use in most of our applications. The Froude number in the prototype is equal to 0.45.

Now, if we use water in the model, then our density and the viscosity between the model and the prototype are the same. If we select a model depth of 0.5, now let's just completely arbitrate in this case to get the problem going because you have to make an assumption of a starting point. We're gonna have to make the Froude similarity for the model to give a velocity of 1.58 feet per second. However, that's holding Froude similarity. If we go into Reynolds similarity, you can see in our equation that if we calculate a Reynolds number that is equivalent--or we use the Reynolds number that's equivalent to the prototype, we have a velocity of the model equal to 158 feet per second. You see the problem that we have? We cannot have both Froude and Reynolds similarity and have geometric similarity all in the same situation.

Okay, we have to have some concessions to reality. It's theoretically desirable to have both Reynolds and Froude number similarity. However, we cannot have both Reynolds and Froude number similarity in most cases as I've shown you in this example. Most of our model studies are generally limited to those cases where the effect of viscosity can be neglected, therefore we're not gonna worry about the Reynolds number similarity. Therefore, or as such, we're going to have

the Froude similarity hold. And so we're gonna allow Froude similarity to drive how we construct our model. It's relatively easy to achieve. The models with Froude similarity are ideal for rapidly varied flow problems where gravity forces dominate. Most of these cases, this is dams, culverts, contracted openings, those kinds of problems.

That concludes our session. I would encourage you now to go to the next section, which is going to be an example problem of determining the size of a scale model, where we try to maintain Froude similarity given a certain geometrics similarity.