Specific Energy Concepts

SPEAKER Bob Holmes: All right, in this section of the Lesson 11, we're going to be talking about specific energy and the derivation of how things put together in a particular equations. That's important to know is we looked at the energy equation from Lecture 4, we had the velocity head which was the mean velocity squared. Sometimes we're going to put in the energy correction coefficient or Coriolis coefficient alpha divided by two times the acceleration of gravity plus the potential energy plus the pressure potential which we now usually use as the depth. Now because we have the potential energy concept or z in there, we would have a totally different total energy at sea level than we would in the Colorado Mountains. However, the streams may behave exactly the same in terms of velocity and their depths, so we want to be able to relate them, put them on a common plane and so we introduce the concept of specific energy where we have the energy that we compute "E" is referenced to the bottom of the channel. So if you were on the Atlantic seaboard or if you were in Wyoming or Colorado, we can compare apple to apples when we talk about specific energy.

Now, energy per pound of water at any section of a channel may should respect to the bottom of the channel is the specific energy. We compute that as the depth d, plus alpha, which is the Coriolis coefficient times the mean velocity squared, divided by two times the acceleration of gravity. Again, this excludes the potential energy z. Now because of continuity which is the volumetric discharge equals the cross sectional area times the mean velocity v, we can restate the specific energy equation to say that specific energy E is equal to the depth d, plus alpha Q squared divided by two times the acceleration of gravity times the cross sectional area squared.

Now for a given channel cross section geometry and discharge, the specific energy is a function of depth only. Now let me restate that. If we have a discharge volumetrically that we have in particular channel and we know that a channel has a particular fixed shape, we can draw a specific energy diagram related to the depth and that's only function is that of the depth. If you look on your chart and based on the slide that's in front of you, you'll see this plotted where we have depth plotted along the vertical axis and specific energy E plotted along the x-axis. And you can see that we have a line that comes asymptotically from a 45, down through a minimum point

at the specific energy and then starts to increase again. Now if we go through exercise, we can actually compute specific energy.

If you'll look at the channel, we've got a mean depth or we have a mean width of 10 feet. We have a discharge of 100 cubic feet per second and we assume that our Coriolis coefficient is one. If we go through and assume in a rectangular channel, we can take various depths, plug that into the specific energy equation that you have at the bottom of your screen and then compute the specific energy itself, E. I go through when I assume the value of 0.1. I calculate the specific energy of 155.1. I have a value of depth of 0.5. I get a specific energy of 6.7. I have a depth of 1; specific energy of 2.55, so on and so forth. Now you'll note as I get around 1.4, 1.46 and 2, I've slowed down my incrementations simply because I know that I'm approaching the minimum value of E and I wanna be able to define that part of the curve. Later on in your textbook, you're going to work an example problem or you're going to work a problem in the back that we're going to ask you to actually compute the specific energy and look for that minimum specific energy. That specific energy at that point is going to be something very crucial later on in this lecture. We're going to talk about critical depth and the minimum specific energy. Those two quantities correspond.

Now, I also want to mention that in this concept we have alternate depths. You will note that for one particular energy, specific energy that is, there'll be two depths that satisfy this equation. On your graph, that's noted D1 and D2. The only place that you don't have alternate depths is again at this minimum specific energy location which has a critical value of the depth. Now, if you look at the graph, you'll see that c is where the minimum specific energy for the section at this flow resides. As we will mention later, c corresponds to this critical state of flow and the depth that corresponds to c is known as the critical depth. We often abbreviate that D sub-c. Now when the depth of your flow is greater than the critical depth, the velocity at the end is less than the critical velocity. You can reason this out yourself if you use the continuity equation which is a volumetric discharge Q is equal to cross sectional area times v. This is the range we call subcritical flow and in a slide in a second, we'll actually show you the difference between the two ranges of subcritical

flow and the next range which is the supercritical flow range, that's when the depth is less than the critical depth and that the velocity is greater than the critical velocity.

Now the critical state of flow is defined as the state of flow in which a specific energy is a minimum for a given discharge. Now, remember from before we had that the specific energy E is equal to the depth plus the alpha which is the Coriolis coefficient times the discharge value squared, divided by two times the acceleration of gravity times the cross sectional area squared. Now I'm not going to make you go through the pain of differentiating this, I will simply give you a differential which is a calculus term, but in your notes or in fact on the screen now, you'll see how I go through and take the differential of the specific energy with respect to the depth. I can just simply say for this purpose in the lecture that the differential width of the specific energy with respect to "d" is equal to one minus the velocity squared divided by the acceleration of gravity times the cross sectional area divided by the top width T. Now this cross sectional area divided by the top width is something that we're going to call the hydraulic depth, A/T and we often abbreviate that d. And again, I told you on an earlier lecture, if you had a wide rectangular cross section, we often assume that the hydraulic radius is equal to the depth. Here we're going to have the hydraulic depth being the area divided by the top width. They're slightly different concepts but you get the idea.

Now we also note that the value in the second side of that equation on the right of the equation where we have the differential of the specific energy with respect to depth, we have a value of v squared over g times the area over the top width which again we've talked about as a hydraulic depth. You'll remember from the lecture we had in Lecture 3 that that's actually the square of the Froude number, so we can say that the differential of the specific energy with respect to depth is equal to 1 minus the square root of the Froude number. Now, excuse me, the Froude number squared, not the square of the Froude number. Anyway, I want you to remember that the Froude number as we've defined it earlier was the velocity divided by the square root of gd. Now if I rearrange the differential and recognize from our specific energy curve that that the critical state flow which is point c, that slope which is all differential of the specific energy with respect to d is that's where "dE" "dd" is equal to zero. That's a slope term. We can say now that we plugged

instead of the differential with respect to of E with respect to d, we can plug in the value for zero. That simplifies our equation and we will note that our velocity head which is v sub-c squared over 2g is equal to one half the depths there. So we can say that the critical state of flow, the velocity head is equal to half the hydraulic depth. Note again that this D sub-c is the critical hydraulic depth. That would be the area divided by the top width.

Now if we look back at the formulation that we first had when we differentiate the specific energy with respect to d, you'll note that we did have the Froude number, I mentioned that to you awhile ago, is that the Froude number was equal to v squared over g times the hydraulic depth. Well, again if we have zero for the value of the differential of the specific energy with respect to d, we can now say that the Froude number squared is equal to 1. Taking the square root of both sides, we can say that the Froude number is equal to 1 at this location. If you recall, in the earlier Lecture 3 we talked about that we use the Froude number to determine the difference between supercritical and subcritical flow. When the Froude number is equal to 1, that's the critical state, if it's above 1, we have a supercritical state and below 1, it's a subcritical state. Now, we note that if we substitute this critical state of flow into this, we can assume that the critical velocity is equal to the square root of g times the hydraulic depth at this critical location.

Now, if you remember from your physics textbook, you recall that the square root of the acceleration of gravity times the depth is something we call a celerity wave, the velocity of the celerity wave or gravity wave. This would be the same thing as if you dropped a pebble of water in a quiescent pond as that those waves moved away from where you disturbed the surface that in itself is a celerity wave. That velocity is equal to the acceleration of gravity times the depth of whatever the pond would be, taking the square root of that would be that velocity. Now, you will note that if you're out there in the channel and you were to drop a pebble in and you had subcritical flow, the velocity would not be great enough where--your velocity would not always wash the celerity wave way, you would have some movement upstream of that celerity wave. However if your velocity was greater than the critical velocity, you would not be able to have that celerity wave move upstream. That's how concept we have when we talk about control features when we have a gauge that we're trying to establish a rating or do a step back water. That's why

we always start on the downstream end and work our way up because in subcritical flow you can have impacts felt upstream that occurred downstream. That's the whole concept of step back water computations. If you were to do a step back water computation on a supercritical situation, you would work from upstream to downstream. Although it's not backwater in that case, right, it would be, what, "fore water"? Anyway, that's basically the concept and we'll show that in a laboratory demonstration in a second.

That's going to conclude our lecture for now in this particular section. I encourage you now to go to the flume demonstration that will show you, where we'll show you the difference between subcritical and supercritical flow.