Velocity Profile in Laminar Flow

SPEAKER Rick Huizinga: In this section, we're gonna talk about laminar flow. Generally, laminar flow does not occur in open channels, except when depths are very small, such as when flow is on a parking lot or at the upper end of a watershed where water is flowing through the grass.

At the lower part of your screen is a figure that shows a free-body diagram of a block of water flowing in laminar flow on a shallow surface. We're gonna make the assumption that the slope is very small, that the flow has a unit weight of gamma, that the flow is laminar, and that there is a width, W, into and out of the screen. We're going to look just at the block of water designated by a, b, c, d. And you can see that flow at the depth of D and the bottom of the block is at a distance Y from the bed. On that block are four forces that are acting. The free body, the hydrostatic force acting on the upstream end, F1; the hydrostatic force acting on the downstream, F2; the weight force, which has a small component in the direction of the slope; and the shear stress along the bottom of that block.

Because flow is uniform, we can sum the forces along a slope and set them equal to zero. So we see that F1 plus the weight force times the sine of theta minus the hydrostatic force on the downstream end minus the quantity tau times the length times the width is all equal to zero. The first term is the hydrostatic force on the upstream end. The weight and times the sine of theta is the weight component that's acting along the slope. F2 is the hydrostatic force acting on the downstream end, and the tau, LW term, is the shear force along the bottom of the block.

Because flow is uniform, we can make the assumption that F1 is equal to F2 because the height of the block does not change, and the weight of the block is equal to the term gamma times the quantity D minus Y times L times W. And the component of the weight that's acting along the slope then is gamma times the quantity D minus Y, L times W times the sine of theta.

Well, if we substitute in the weight term, we get the equation F1 minus gamma times the quantity D minus Y, L times W times the sine of theta minus F2 minus tau LW, and that's all equal to zero. Flow is uniform, and so F1 and F2 are equal, and we can cancel them out of the equation. If we rearrange and move some of the terms to the right-hand side, we get gamma times the quantity D minus Y, L times W times the sine of theta is all equal to tau times L times W.

And now because L and W are on both sides of the equation, we can cancel them out as well. So we can simply and we get the equation tau minus--and we get the equation tau is equal to gamma times the quantity D minus Y times the sine of theta. And this is called the universal shear stress equation, and it's valid for either laminar or turbulent flow.

Now, recall from Lecture 1 and earlier in this lecture, that the shear force, tau, is equal to mu times the quantity DVDY. So let's set tau is equal to mu times DVDY equal to the universal shear stress, gamma times the quantity D minus Y sine of theta. If we rearrange and just leave the DY on the left-hand side, we get that DV is equal to gamma over mu times the quantity D minus Y sine of theta DY.

And if we integrate that over the depth from the bed to the top surface, D, we get the velocity is equal to gamma over mu times sine of theta times the interval of D minus Y, DY. And when we integrate that the result is that the velocity is equal to gamma over mu times the sine of theta times the quantity D times Y minus Y squared over two plus the constant of integration, C. Because the velocity is equal to zero when the distance above the bed is equal to zero, then the constant of integration also is zero. So what we have then is the velocity distribution equation for laminar open channel flow. And that is equal to--V is equal to the quantity gamma over mu times the sine of theta times the quantity D times Y minus Y squared over two. And this is a parabolic equation, which means that the velocity profile will have a parabolic shape, as shown in the image on the right-hand side of the screen.

We can also compute a unit discharge and a mean velocity from the universal velocity equation. The discharge per unit width or the unit discharge is found by integrating again and we wind up with the equation on the right-hand side, in the middle of the screen, little "q," which is the unit discharge, is equal to gamma over mu times the sine of theta times D cubed over three.

And the mean velocity then is found by dividing the unit discharge by the cross-sectional area, D times one. The mean discharge is generally designated with a bar over the capital letter V, and we call that V bar. And that is equal to gamma over mu times the sine of theta times D squared over three.

And that is the end of the laminar flow discussion section.