Velocity Profile in Turbulent Flow

SPEAKER Rick Huizinga: In this section, we're going to talk about turbulent flow, which occurs in an overwhelming majority of open channel flow. Recall the universal shear stress equation from the previous section, we derived that in our discussion about laminar flow and that was equal to tau is equal to gamma times the quantity D minus Y times the sine of theta. D is the depth of flow. Y is the distance above the bed.

In laminar flow, tau is a function of the dynamic viscosity, mu. However, in turbulent flow, random particle movement causes additional shear stress, which is induced by the momentum transfer, such that the total shear stress in the fluid is a combination of the viscous shear stress and the turbulent shear stress. And so the total shear stress is shown there on the right-hand side of the screen is tau is equal to mu over DVDY plus a turbulent component of shear stress.

As one moves away from the flow boundary, the turbulent forces drastically outweigh the viscous forces of shear stress. And so, for the most part, in turbulent flow, the shear stress is entirely the result of turbulence.

Now, Prandtl developed a theory to describe the shear stress cause by turbulence, and he described it as rho times L squared times DV over DY squared. And the L is a mixing length, the distance that each particle moves from its mean position. If flow is occurring in a section that is this deep, the mixing length would be the distance that an individual particle of flow would travel upward or downward. It's very small at the bed and near the surface of the flow, but it tends to be larger in the middle of the flow.

Now, the equation for mixing length that Prandtl developed is shown now in the center of your screen. L is equal to kappa times Y times the square root of one minus Y over D. And this Greek kappa is the von Karman constant and is generally taken as the quantity 0.4.

Well, if we combine our universal shear stress equation and the turbulence shear stress equation developed by Prandtl and then rearrange them, we get the equation shown in the middle of your screen, rho L squared over--we get the equation in the middle of your screen, rho times L squared times DV over DY squared, and that is equal to gamma times the quantity D minus Y times the sine of theta.

Now, if we substitute the length, the mixing length equation in, we then get this longer equation at the bottom of the screen, rho times kappa squared times Y squared times the quantity one minus Y over D times DV over DY squared, and that's all equal to gamma times the quantity D minus Y times the sine of theta. I move that to the top of the screen because we're going to do some mathematics on these equations.

If we multiply the right side of the equation by D over D, we can do some interesting mathematics with this, and it's perfectly acceptable to do this because that's multiplying by one. If we rearrange our quantities and move the DVDY by itself on the left-hand side, we then get on the right-hand side, D times gamma times the quantity one minus Y over D times the sine of theta all over rho times kappa squared Y squared times the quantity one minus Y over D.

And if we simplify then by taking the square root of both sides, we get that DVDY--we get that DV over DY is equal to the square root of gamma times D times the sine of theta over rho times the quantity one over kappa times Y.

Now remember, our universal shear stress equation, tau is equal to gamma times D minus Y times the sine of theta. At the bed, the distance above the bed is zero. And so tau zero is equal to the quantity Y times D times the sine of theta (**Note:** the narrator misspeaks here and says "Y" when in fact it is the unit weight "gamma.") And this is a new term, this tau sub-zero. This is the shear stress at the bed, and you will see this in future lectures where we talk about the bed shear stress. We substitute that into the equation at the upper right, we get the equation in the middle of the screen and that is DV over DY is equal to the square root of tau zero over rho times the quantity one over kappa times Y.

And I like to introduce the second term, U star, which is the quantity of the square root of tau zero over rho. This is called the shear velocity or the friction velocity, and it has dimensions of velocity feet over--feet per second. If we substitute that into our equation and rearrange, we get that DV is equal to U star over kappa times Y times DY. Then if we integrate that, we are able to get the Prandtl-von Karman Universal Velocity Distribution Law for Turbulent Flow, which is V is equal to U star over kappa times the natural log of Y over Y zero. Y zero is the constant of integration of

the integration. And it's physically equal to Y when V is equal to zero. For values of Y that are less than Y zero, flow is in the laminar range, and kappa, again, is the von Karman constant of 0.4.

Now we're able to determine the unit discharge and mean velocity for turbulent flow just like we did for laminar flow, the discharge per unit width or unit discharge is found by integrating on V, which gives us the equation on the right-hand side of the screen, Q is equal to U star times D over kappa times the natural log of D over the quantity E times Y zero. And that E term is the base of the natural logarithms, 2.718. The mean velocity is found by dividing the unit discharge by the cross-sectional area, D times one. So, again, we get V bar is equal to U star over kappa times the natural log of D over the quantity E times Y zero.

Now it's interesting that we can compute the mean velocity in the vertical from this equation when Y, the distance above the bed, is set equal to the quantity D over E, we see that the individual point velocity, U star over kappa times a natural log of D over E times the Y sub-zero is equal to the mean velocity. So the quantity D over E, if we plug in E is equal to 2.718, we get that D over E is 0.368 times D.

Well, recall that Y is measured up from the bed. So the point velocity that is equal to the average velocity occurs at a depth from the surface of one minus D over E, or 0.632 D. If you've ever taken a discharge measurement and you wanna do an average velocity at a single point, you take it at 0.6 times the depth, and that is how that quantity was arrived at for a single point velocity.

Now, we need to determine the constant of integration, Y zero. And as it turns out, it's different for smooth surfaces than it is for rough surfaces. For a smooth boundary, it's been experimentally derived that Y sub-zero is equal to V over the quantity nine times U star. So if we plug that into our equation, you see that V is equal to U star over kappa times the natural log of nine U star times Y over V.

Now if we use a quantity of 0.4 for kappa and convert it to common logarithms, our velocity equation becomes 5.75 times U star times the log of nine U star times Y times V over V. (**Note**: the narrator misspeaks here and calls this "V" when in fact it is the kinematic viscosity, "nu.")

For rough surfaces, the value of Y sub-zero that's been experimentally derived is equal to the quantity K sub-s over 30. And K sub-s is the effective height of irregularities on the surface. So, if we substitute that into our velocity equation, we get U star over kappa times the natural log of 30 Y over K sub-s. And again, if we use 0.4 for kappa and convert to common logarithms, our velocity equation becomes 5.75 times U star times the log of 30 Y over K sub-s.

And that is the end of the turbulent flow section.