UNITED STATES
DEPARTMENT OF THE INTERIOR
FRED A. SEATON, Secretary
BUREAU OF LAND MANAGEMENT
EDWARD WOOZLEY, Director

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STANDARD FIELD TABLES
AND
TRIGONOMETRIC FORMULAS

A Supplement to the Manual of Instructions
for the Survey of the Public Lands
of the United States

EIGHTH EDITION 1956
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TABLE 11.—CONVERGENCY OF MERIDIANS, SIX MILES LONG AND SIX MILES APART, AND DIFFERENCES OF LATITUDE AND LONGITUDE.

Lat.	Convergency.		Difference of longitude per range.		Difference of latitude for—	
	On the parallel.	Angle.	In arc.	In time.	1 mi.	1 Tp.
°	*Lks.*	′ ″	′ ″	*Seconds.*		
25	33.9	2 25	5 44.34	22.96		
26	35.4	2 32	5 47.20	23.15		
27	37.0	2 39	5 50.22	23.35	0.871	5.229
28	38.6	2 46	5 53.40	23.56		
29	40.2	2 53	5 56.74	23.78		
30	41.9	3 0	6 0.26	24.02		
31	43.6	3 7	6 3.97	24.26		
32	45.4	3 15	6 7.87	24.52	0.871	5.225
33	47.2	3 23	6 11.96	24.80		
34	49.1	3 30	6 16.26	25.08		
35	50.9	3 38	6 20.78	25.39		
36	52.7	3 46	6 25.53	25.70		
37	54.7	3 55	6 30.52	26.03	0.870	5.221
38	56.8	4 4	6 35.76	26.38		
39	58.8	4 13	6 41.27	26.75		
40	60.9	4 22	6 47.06	27.14		
41	63.1	4 31	6 53.15	27.54		
42	65.4	4 41	6 59.56	27.97	0.869	5.216
43	67.7	4 51	7 6.29	28.42		
44	70.1	5 1	7 13.39	28.89		
45	72.6	5 12	7 20.86	29.39		
46	75.2	5 23	7 28.74	29.92		
47	77.8	5 34	7 37.04	30.47	0.869	5.211
48	80.6	5 46	7 45.80	31.05		
49	83.5	5 59	7 55.05	31.67		
50	86.4	6 12	8 4.83	32.32		
51	89.6	6 25	8 15.17	33.03		
52	92.8	6 39	8 26.13	33.74	0.868	5.207
53	96.2	6 54	8 37.75	34.52		
54	99.8	7 9	8 50.07	35.34		
55	103.5	7 25	9 3.18	36.22		
56	107.5	7 42	9 17.12	37.14		
57	111.6	8 0	9 31.97	38.13	0.867	5.202
58	116.0	8 19	9 47.83	39.19		
59	120.6	8 38	10 4.78	40.32		
60	125.5	8 59	10 22.94	41.52		
61	130.8	9 22	10 42.42	42.83		
62	136.3	9 46	11 3.38	44.22	0.866	5.198
63	142.2	10 11	11 25.97	45.73		
64	148.6	10 38	11 50.37	47.36		
65	155.0	11 8	12 16.82	49.12		
66	162.8	11 39	12 45.55	51.04		
67	170.7	12 13	13 16.88	53.12	0.866	5.195
68	179.3	12 51	13 51.15	55.41		
69	188.7	13 31	14 28.77	57.92		
70	199.1	14 15	15 10.26	60.68	0.866	5.193

TABLE 12.—AZIMUTHS OF THE TANGENT TO THE PARALLEL.

Lat.	1 mi.	2 mi.	3 mi.	4 mi.	5 mi.	6 mi.
°	° ′	° ′	° ′	° ′	° ′	° ′
25	89 59.6	89 59.2	89 58.8	89 58.4	89 58.0	89 57.6
26	59.6	59.2	58.7	58.3	57.9	57.5
27	59.6	59.1	58.7	58.2	57.8	57.4
28	59.5	59.1	58.6	58.2	57.7	57.2
29	59.5	59.0	58.6	58.1	57.6	57.1
30	59.5	59.0	58.5	58.0	57.5	57.0
31	59.5	59.0	58.4	57.9	57.4	56.9
32	59.5	58.9	58.4	57.8	57.3	56.8
33	59.4	58.9	58.3	57.7	57.2	56.6
34	59.4	58.8	58.2	57.7	57.1	56.5
35	59.4	58.8	58.2	57.6	57.0	56.4
36	59.4	58.7	58.1	57.5	56.9	56.2
37	59.3	58.7	58.0	57.4	56.7	56.1
38	59.3	58.6	58.0	57.3	56.6	55.9
39	59.3	58.6	57.9	57.2	56.5	55.8
40	59.3	58.5	57.8	57.1	56.4	55.6
41	59.2	58.5	57.7	57.0	56.2	55.5
42	59.2	58.4	57.7	56.9	56.1	55.3
43	59.2	58.4	57.6	56.8	56.0	55.2
44	59.2	58.3	57.5	56.7	55.8	55.0
45	59.1	58.3	57.4	56.5	55.7	54.8
46	59.1	58.2	57.3	56.4	55.5	54.6
47	59.1	58.1	57.2	56.3	55.4	54.4
48	59.0	58.1	57.1	56.2	55.2	54.2
49	59.0	58.0	57.0	56.0	55.0	54.0
50	59.0	57.9	56.9	55.9	54.8	53.8
51	58.9	57.9	56.8	55.7	54.6	53.6
52	58.9	57.8	56.7	55.6	54.5	53.4
53	58.8	57.7	56.6	55.4	54.2	53.1
54	58.8	57.6	56.4	55.2	54.0	52.8
55	58.8	57.5	56.3	55.1	53.8	52.6
56	58.7	57.4	56.2	54.9	53.6	52.3
57	58.7	57.3	56.0	54.7	53.3	52.0
58	58.6	57.2	55.8	54.5	53.1	51.7
59	58.6	57.1	55.7	54.2	52.8	51.4
60	58.5	57.0	55.5	54.0	52.5	51.0
61	58.4	56.9	55.3	53.8	52.2	50.6
62	58.4	56.7	55.1	53.5	51.9	50.2
63	58.3	56.6	54.9	53.2	51.5	49.8
64	58.2	56.5	54.7	52.9	51.1	49.4
65	58.1	56.3	54.4	52.6	50.7	48.9
66	58.1	56.1	54.2	52.2	50.3	48.4
67	58.0	55.9	53.9	51.8	49.8	47.8
68	57.9	55.7	53.6	51.4	49.3	47.2
69	57.8	55.5	53.2	51.0	48.8	46.5
70	89 57.6	89 55.3	89 52.9	89 50.5	89 48.1	89 45.8

TABLE 13.—OFFSETS, IN LINKS, FROM THE TANGENT TO THE PARALLEL.

Lat.	½ mi.	1 mi.	1½ mi.	2 mi.	2½ mi.	3 mi.	3½ mi.	4 mi.	4½ mi.	5 mi.	5½ mi.	6 mi.
°												
25	0	0	1	2	3	4	6	8	10	12	14	17
26	0	0	1	2	3	4	6	8	10	12	15	18
27	0	1	1	2	3	5	6	8	10	13	16	18
28	0	1	1	2	3	5	7	9	11	13	16	19
29	0	1	1	2	3	5	7	9	11	14	17	20
30	0	1	1	2	4	5	7	9	12	15	18	21
31	0	1	1	2	4	5	7	10	12	15	18	22
32	0	1	1	3	4	6	8	10	13	16	19	23
33	0	1	1	3	4	6	8	10	13	16	20	24
34	0	1	2	3	4	6	8	11	14	17	21	25
35	0	1	2	3	4	6	8	11	14	18	21	25
36	0	1	2	3	5	7	9	12	15	18	22	26
37	0	1	2	3	5	7	9	12	15	19	23	27
38	0	1	2	3	5	7	10	13	16	20	24	28
39	0	1	2	3	5	7	10	13	17	20	25	29
40	0	1	2	3	5	8	10	14	17	21	26	30
41	0	1	2	4	5	8	11	14	18	22	27	32
42	0	1	2	4	6	8	11	15	18	23	27	33
43	0	1	2	4	6	8	11	15	19	24	28	34
44	0	1	2	4	6	9	12	16	20	24	29	35
45	0	1	2	4	6	9	12	16	20	25	30	36
46	0	1	2	4	7	9	13	17	21	26	32	38
47	0	1	2	4	7	10	13	17	22	27	33	39
48	0	1	3	4	7	10	14	18	23	28	34	40
49	0	1	3	5	7	10	14	19	23	29	35	42
50	0	1	3	5	8	11	15	19	24	30	36	43
51	0	1	3	5	8	11	15	20	25	31	38	45
52	0	1	3	5	8	12	16	21	26	32	39	46
53	0	1	3	5	8	12	16	21	27	33	40	48
54	0	1	3	6	9	12	17	22	28	35	42	50
55	0	1	3	6	9	13	18	23	29	36	43	52
56	0	1	3	6	9	13	18	24	30	37	45	54
57	0	2	3	6	10	14	19	25	31	39	47	56
58	0	2	4	6	10	14	20	26	33	40	49	58
59	0	2	4	7	10	15	20	27	34	42	51	60
60	0	2	4	7	11	16	21	28	35	44	53	63
61	0	2	4	7	11	16	22	29	37	45	55	65
62	0	2	4	8	12	17	23	30	38	47	57	68
63	0	2	4	8	12	18	24	32	40	49	60	71
64	1	2	5	8	13	19	25	33	42	52	62	74
65	1	2	5	9	13	19	26	34	44	54	65	78
66	1	2	5	9	14	20	28	36	46	57	68	81
67	1	2	5	9	15	21	29	38	48	59	72	85
68	1	2	6	10	16	22	30	40	50	62	75	90
69	1	3	6	10	16	24	32	42	53	66	79	94
70	1	3	6	11	17	25	34	44	56	69	84	100

TABLE 14.—AZIMUTHS OF THE SECANT.

Lat.	0 mi.	1 mi.	2 mi.	3 mi.	Deflection angle 6 mi.
°	° ′	° ′	° ′		′ ″
25	89 58.8	89 59.2	89 59.6	90°	2 25
26	58.7	59.2	59.6	E or W.	2 32
27	58.7	59.1	59.6	" " "	2 39
28	58.6	59.1	59.5	" " "	2 46
29	58.6	59.0	59.5	" " "	2 53
30	58.5	59.0	59.5	" " "	3 0
31	58.4	59.0	59.5	" " "	3 7
32	58.4	58.9	59.5	" " "	3 15
33	58.3	58.9	59.4	" " "	3 23
34	58.2	58.8	59.4	" " "	3 30
35	58.2	58.8	59.4	" " "	3 38
36	58.1	58.7	59.4	" " "	3 46
37	58.0	58.7	59.3	" " "	3 55
38	58.0	58.6	59.3	" " "	4 4
39	57.9	58.6	59.3	" " "	4 13
40	57.8	58.5	59.3	" " "	4 22
41	57.7	58.5	59.2	" " "	4 31
42	57.7	58.4	59.2	" " "	4 41
43	57.6	58.4	59.2	" " "	4 51
44	57.5	58.3	59.2	" " "	5 1
45	57.4	58.3	59.1	" " "	5 12
46	57.3	58.2	59.1	" " "	5 23
47	57.2	58.1	59.1	" " "	5 34
48	57.1	58.1	59.0	" " "	5 46
49	57.0	58.0	59.0	" " "	5 59
50	56.9	57.9	59.0	" " "	6 12
51	56.8	57.9	58.9	" " "	6 25
52	56.7	57.8	58.9	" " "	6 39
53	56.6	57.7	58.8	" " "	6 54
54	56.4	57.6	58.8	" " "	7 9
55	56.3	57.5	58.8	" " "	7 25
56	56.2	57.4	58.7	" " "	7 42
57	56.0	57.3	58.7	" " "	8 0
58	55.8	57.2	58.6	" " "	8 19
59	55.7	57.1	58.6	" " "	8 38
60	55.5	57.0	58.5	" " "	8 59
61	55.3	56.9	58.4	" " "	9 22
62	55.1	56.7	58.4	" " "	9 46
63	54.9	56.6	58.3	" " "	10 11
64	54.7	56.5	58.2	" " "	10 38
65	54.4	56.3	58.1	" " "	11 8
66	54.2	56.1	58.1	" " "	11 39
67	53.9	55.9	58.0	" " "	12 13
68	53.6	55.7	57.9	" " "	12 51
69	53.2	55.5	57.8	" " "	13 31
70	89 52.9	89 55.3	89 57.6	" " "	14 15
	6 mi.	5 mi.	4 mi.	3 mi.	

TABLE 15.—OFFSETS, IN LINKS, FROM THE SECANT TO THE PARALLEL.

Lat.	0 mi.	½ mi.	1 mi.	1½ mi.	2 mi.	2½ mi.	3 mi.
°							
25	2 N.	1 N.	0	1 S.	1 S.	2 S.	2 S.
26	2	1	0	1	1	2	2
27	3	1	0	1	2	2	2
28	3	1	0	1	2	2	2
29	3	1	0	1	2	2	2
30	3	1	0	1	2	2	2
31	3	1	0	1	2	2	2
32	3	1	0	1	2	2	3
33	3	1	0	1	2	2	3
34	3	2	0	1	2	3	3
35	4	2	0	1	2	3	3
36	4	2	0	1	2	3	3
37	4	2	0	1	2	3	3
38	4	2	0	1	2	3	3
39	4	2	0	1	2	3	3
40	4	2	0	1	3	3	3
41	4	2	0	2	3	3	4
42	5	2	0	2	3	3	4
43	5	2	0	2	3	4	4
44	5	2	0	2	3	4	4
45	5	2	0	2	3	4	4
46	5	2	0	2	3	4	4
47	5	2	0	2	3	4	4
48	6	3	0	2	3	4	4
49	6	3	0	2	3	4	5
50	6	3	0	2	4	4	5
51	6	3	0	2	4	5	5
52	6	3	0	2	4	5	5
53	7	3	0	2	4	5	5
54	7	3	0	2	4	5	6
55	7	3	0	3	4	5	6
56	7	3	0	3	4	6	6
57	8	3	0	3	5	6	6
58	8	4	0	3	5	6	6
59	8	4	0	3	5	6	7
60	9	4	0	3	5	7	7
61	9	4	0	3	5	7	7
62	9	4	0	3	6	7	8
63	10	4	0	3	6	7	8
64	10	5	0	4	6	8	8
65	11	5	0	4	6	8	9
66	11	5	0	4	7	8	9
67	12	5	0	4	7	9	9
68	12	6	0	4	7	9	10
69	13	6	0	5	8	10	10
70	14 N.	6 N.	0	5 S.	8 S.	10 S.	11 S.
	6 mi.	5½ mi.	5 mi.	4½ mi.	4 mi.	3½ mi.	3 mi.

TABLE 16.—LENGTHS OF ARCS OF THE EARTH'S SURFACE.

Lengths of degrees of the parallel.				Lengths of degrees of the meridian.	
Lat.	Statute miles.	Lat.	Statute miles.	Lat.	Statute miles.
° ′		° ′		°	
25 0	62.729	47 30	46.818	25	68.829
30	62.473	48 0	46.372	26	68.839
26 0	62.212	30	45.922	27	68.848
30	61.946	49 0	45.469	28	68.858
27 0	61.676	30	45.012	29	68.869
30	61.401	50 0	44.552	30	68.879
28 0	61.122	30	44.088	31	68.890
30	60.837	51 0	43.621	32	68.901
29 0	60.548	30	43.150	33	68.912
30	60.254	52 0	42.676	34	68.923
30 0	59.956	30	42.199	35	68.935
30	59.653	53 0	41.719	36	68.946
31 0	59.345	30	41.235	37	68.958
30	59.033	54 0	40.749	38	68.969
32 0	58.716	30	40.259	39	68.981
30	58.396	55 0	39.766	40	68.993
33 0	58.071	30	39.270	41	69.006
30	57.741	56 0	38.771	42	69.018
34 0	57.407	30	38.269	43	69.030
30	57.068	57 0	37.764	44	69.042
35 0	56.725	30	37.256	45	69.054
30	56.378	58 0	36.745	46	69.066
36 0	56.027	30	36.232	47	69.079
30	55.671	59 0	35.716	48	69.091
37 0	55.311	30	35.196	49	69.103
30	54.947	60 0	34.674	50	69.115
38 0	54.579	30	34.150	51	69.127
30	54.206	61 0	33.623	52	69.139
39 0	53.829	30	33.093	53	69.151
30	53.448	62 0	32.560	54	69.163
40 0	53.063	30	32.025	55	69.175
30	52.674	63 0	31.488	56	69.186
41 0	52.281	30	30.948	57	69.197
30	51.884	64 0	30.406	58	69.209
42 0	51.483	30	29.862	59	69.220
30	51.078	65 0	29.315	60	69.230
43 0	50.669	30	28.766	61	69.241
30	50.257	66 0	28.215	62	69.251
44 0	49.840	30	27.661	63	69.261
30	49.419	67 0	27.106	64	69.271
45 0	48.995	30	26.548	65	69.281
30	48.567	68 0	25.988	66	69.290
46 0	48.136	30	25.426	67	69.299
30	47.700	69 0	24.862	68	69.308
47 0	47.261	30	24.297	69	69.316
47 30	46.818	70 0	23.729	70	69.324

The lengths of degrees of the meridian are tabulated to correspond to the length of the arc of which the tabulated latitude is the middle, thus the quantity 68.993, opposite latitude 40° 0′ is the number of miles between latitude 39° 30′ and 40° 30′.

The above tabie is an abridgment of a table published by the U. S. Coast and Geodetic Survey based on the values of the Clarke spheroid.

TABLE 17.—APPARENT TIME OF SUNSET FOR N. DECLINATIONS, OR SUNRISE FOR S. DECLINATIONS.

Sun's Decl.	Latitude.				
	25°	30°	35°	40°	45°
° ′	h m	h m	h m	h m	h m
0 0	6 0	6 0	6 0	6 0	6 0
1	6 2	6 2	6 3	6 3	6 4
2	6 4	6 5	6 6	6 7	6 8
3	6 6	6 7	6 8	6 10	6 12
4	6 7	6 9	6 11	6 13	6 16
5	6 9	6 12	6 14	6 17	6 20
6	6 11	6 14	6 17	6 20	6 24
7	6 13	6 16	6 20	6 24	6 28
8	6 15	6 19	6 23	6 27	6 32
9	6 17	6 21	6 25	6 31	6 36
10	6 19	6 23	6 28	6 34	6 41
11	6 21	6 26	6 31	6 38	6 45
12	6 23	6 28	6 34	6 41	6 49
13	6 25	6 31	6 37	6 45	6 53
14	6 27	6 33	6 40	6 48	6 58
15	6 29	6 36	6 43	6 52	7 2
16	6 31	6 38	6 46	6 56	7 7
17	6 33	6 41	6 49	6 59	7 11
18	6 35	6 43	6 53	7 3	7 16
19	6 37	6 46	6 56	7 7	7 21
20	6 39	6 49	6 59	7 11	7 25
21	6 41	6 51	7 2	7 15	7 30
22	6 43	6 54	7 6	7 19	7 35
23	6 46	6 57	7 9	7 23	7 40
23 27	6 47	6 58	7 11	7 25	7 43

Sun's Decl.	Latitude.				
	50°	55°	60°	65°	70°
° ′	h m	h m	h m	h m	h m
0 0	6 0	6 0	6 0	6 0	6 0
1	6 5	6 6	6 7	6 9	6 11
2	6 10	6 11	6 14	6 17	6 22
3	6 14	6 17	6 21	6 26	6 33
4	6 19	6 23	6 28	6 34	6 44
5	6 24	6 29	6 35	6 42	6 56
6	6 29	6 35	6 42	6 52	7 7
7	6 34	6 40	6 49	7 0	7 19
8	6 39	6 46	6 56	7 10	7 31
9	6 44	6 52	7 4	7 19	7 43
10	6 49	6 58	7 11	7 29	7 56
11	6 54	7 4	7 19	7 39	8 9
12	6 59	7 11	7 26	7 48	8 23
13	7 4	7 17	7 34	7 59	8 37
14	7 9	7 23	7 42	8 9	8 53
15	7 14	7 30	7 51	8 20	9 10
16	7 20	7 37	7 59	8 32	9 28
17	7 25	7 44	8 8	8 44	9 48
18	7 31	7 51	8 17	8 57	10 13
19	7 37	7 58	8 26	9 10	10 44
20	7 43	8 5	8 36	9 25	12 00
21	7 49	8 13	8 47	9 42	
22	7 55	8 21	8 58	10 0	
23	8 2	8 29	9 9	10 22	
23 27	8 5	8 33	9 15	10 34	

TABLE 18.—CONVERSION OF DEGREES TO TIME, AND TIME TO DEGREES.

To reduce degrees to time.						To reduce time to degrees.					
°	H. M.	°	H. M.	Degrees.	Hours. Minutes.	Hours.	Degrees.	M.	° ′	M.	° ′
′	M. S.	′	M. S.					S.	′ ″	S.	′ ″
″	S. T.	″	S. T.					T.	″ ‴	T.	″ ‴
1	0 4	51	3 24	101	6 44	1	15	1	0 15	51	12 45
2	0 8	52	3 28	102	6 48	1½	22½	2	0 30	52	13 0
3	0 12	53	3 32	103	6 52	2	30	3	0 45	53	13 15
4	0 16	54	3 36	104	6 56	2½	37½	4	1 0	54	13 30
5	0 20	55	3 40	105	7 0	3	45	5	1 15	55	13 45
6	0 24	56	3 44	106	7 4	3½	52½	6	1 30	56	14 0
7	0 28	57	3 48	107	7 8	4	60	7	1 45	57	14 15
8	0 32	58	3 52	108	7 12	4½	67½	8	2 0	58	14 30
9	0 36	59	3 56	109	7 16	5	75	9	2 15	59	14 45
10	0 40	60	4 0	110	7 20	5½	82½	10	2 30	60	15 0
11	0 44	61	4 4	115	7 40	6	90	11	2 45	61	15 15
12	0 48	62	4 8	120	8 0	6½	97½	12	3 0	62	15 30
13	0 52	63	4 12	125	8 20	7	105	13	3 15	63	15 45
14	0 56	64	4 16	130	8 40	7½	112½	14	3 30	64	16 0
15	1 0	65	4 20	135	9 0	8	120	15	3 45	65	16 15
16	1 4	66	4 24	140	9 20	8½	127½	16	4 0	66	16 30
17	1 8	67	4 28	145	9 40	9	135	17	4 15	67	16 45
18	1 12	68	4 32	150	10 0	9½	142½	18	4 30	68	17 0
19	1 16	69	4 36	155	10 20	10	150	19	4 45	69	17 15
20	1 20	70	4 40	160	10 40	10½	157½	20	5 0	70	17 30
21	1 24	71	4 44	165	11 0	11	165	21	5 15	71	17 45
22	1 28	72	4 48	170	11 20	11½	172½	22	5 30	72	18 0
23	1 32	73	4 52	175	11 40	12	180	23	5 45	73	18 15
24	1 36	74	4 56	180	12 0	12½	187½	24	6 0	74	18 30
25	1 40	75	5 0	185	12 20	13	195	25	6 15	75	18 45
26	1 44	76	5 4	190	12 40	13½	202½	26	6 30	76	19 0
27	1 48	77	5 8	195	13 0	14	210	27	6 45	77	19 15
28	1 52	78	5 12	200	13 20	14½	217½	28	7 0	78	19 30
29	1 56	79	5 16	205	13 40	15	225	29	7 15	79	19 45
30	2 0	80	5 20	210	14 0	15½	232½	30	7 30	80	20 0
31	2 4	81	5 24	215	14 20	16	240	31	7 45	81	20 15
32	2 8	82	5 28	220	14 40	16½	247½	32	8 0	82	20 30
33	2 12	83	5 32	225	15 0	17	255	33	8 15	83	20 45
34	2 16	84	5 36	230	15 20	17½	262½	34	8 30	84	21 0
35	2 20	85	5 40	235	15 40	18	270	35	8 45	85	21 15
36	2 24	86	5 44	240	16 0	18½	277½	36	9 0	86	21 30
37	2 28	87	5 48	245	16 20	19	285	37	9 15	87	21 45
38	2 32	88	5 52	250	16 40	19½	292½	38	9 30	88	22 0
39	2 36	89	5 56	255	17 0	20	300	39	9 45	89	22 15
40	2 40	90	6 0	260	17 20	20½	307½	40	10 0	90	22 30
41	2 44	91	6 4	270	18 0	21	315	41	10 15	91	22 45
42	2 48	92	6 8	280	18 40	21½	322½	42	10 30	92	23 0
43	2 52	93	6 12	290	19 20	22	330	43	10 45	93	23 15
44	2 56	94	6 16	300	20 0	22½	337½	44	11 0	94	23 30
45	3 0	95	6 20	310	20 40	23	345	45	11 15	95	23 45
46	3 4	96	6 24	320	21 20	23½	352½	46	11 30	96	24 0
47	3 8	97	6 28	330	22 0	24	360	47	11 45	97	24 15
48	3 12	98	6 32	340	22 40			48	12 0	98	24 30
49	3 16	99	6 36	350	23 20			49	12 15	99	24 45
50	3 20	100	6 40	360	24 0			50	12 30	100	25 0

TABLE 19.—SIDEREAL CONVERSIONS.

Long.	Hours.	Longitude. 0° 0′	2° 30′	5° 0′	7° 30′	10° 0′	12° 30′	15° 0′
		Minutes. 0	10	20	30	40	50	60
°		m s	m s	m s	m s	m s	m s	m s
0	0	0 0	0 2	0 3	0 5	0 7	0 8	0 10
15	1	0 10	0 11	0 13	0 15	0 16	0 18	0 20
30	2	0 20	0 21	0 23	0 25	0 26	0 28	0 30
45	3	0 30	0 31	0 33	0 34	0 36	0 38	0 39
60	4	0 39	0 41	0 43	0 44	0 46	0 48	0 49
75	5	0 49	0 51	0 53	0 54	0 56	0 57	0 59
90	6	0 59	1 1	1 2	1 4	1 6	1 7	1 9
105	7	1 9	1 11	1 12	1 14	1 15	1 17	1 19
120	8	1 19	1 20	1 22	1 24	1 25	1 27	1 29
135	9	1 29	1 30	1 32	1 34	1 35	1 37	1 38
150	10	1 38	1 40	1 42	1 43	1 45	1 47	1 48
165	11	1 48	1 50	1 52	1 53	1 55	1 56	1 58
180	12	1 58	2 0	2 1	2 3	2 5	2 6	2 8
195	13	2 8	2 10	2 11	2 13	2 15	2 16	2 18
210	14	2 18	2 19	2 21	2 23	2 24	2 26	2 28
225	15	2 28	2 29	2 31	2 33	2 34	2 36	2 37
240	16	2 37	2 39	2 41	2 42	2 44	2 46	2 47
255	17	2 47	2 49	2 51	2 52	2 54	2 56	2 57
270	18	2 57	2 59	3 0	3 2	3 4	3 5	3 7
285	19	3 7	3 9	3 10	3 12	3 14	3 15	3 17
300	20	3 17	3 18	3 20	3 22	3 23	3 25	3 27
315	21	3 27	3 28	3 30	3 32	3 33	3 35	3 37
330	22	3 37	3 38	3 40	3 41	3 43	3 45	3 46
345	23	3 46	3 48	3 50	3 51	3 53	3 55	3 56
	Hours.							

Sidereal into mean solar time: To be subtracted from a sidereal time interval: Argument hours and minutes of sidereal interval.

Mean Solar into sidereal time: To be added to a mean time interval: Argument hours and minutes of mean time interval.

Upper culmination of Polaris: Amount to be subtracted from the Greenwich mean time of upper culmination of Polaris, or of elongation, to obtain the local mean time of upper culmination, or of elongation: Argument longitude west from Greenwich.

The above table is an abridged mean of two tables given in the American Ephemeris and Nautical Almanac for similar conversions; reductions involving a refinement exceeding 0.8 seconds must be made from the more elaborate tables.

TABLE 20.—MEAN REFRACTIONS IN ZENITH DISTANCE.

Bar.=29.6 Ins. Temp.=50° F.

Apparent altitude.	Refraction.	Apparent altitude.	Refraction.	Apparent altitude.	Refraction.
° ′	′ ″	° ′	′ ″	°	′ ″
7 30	6 53	12 0	4 25	25	2 3
40	45	12 30	15	26	1 58
7 50	37	13 0	4 5	27	53
8 0	30	13 30	3 56	28	48
10	22	14 0	47	29	44
20	15	14 30	39	30	40
30	8	15 0	32	32	32
40	6 2	15 30	25	34	25
8 50	5 55	16 0	19	36	19
9 0	49	16 30	13	38	14
10	43	17 0	7	40	9
20	38	17 30	3 1	42	4
30	32	18 0	2 56	44	1 0
40	26	18 30	51	46	0 56
9 50	21	19 0	46	48	52
10 0	16	19 30	42	50	48
20	5 6	20 0	37	55	40
10 40	4 57	21 0	29	60	33
11 0	48	22 0	22	65	27
20	40	23 0	15	70	21
11 40	32	24 0	9	80	10
12 0	4 25	25 0	2 3	90	0 0

SUN'S PARALLAX IN ALTITUDE.

Apparent altitude.....	0°	26°	48°	63°	77°	90°
Sun's parallax	8″.9	8″	6″	4″	2″	0″

TABLE 21.—COEFFICIENTS TO APPLY TO MEAN REFRACTIONS FOR VARIATIONS IN BAROMETER AND TEMPERATURE.

Barometer.	Elevation above sea level.	Coefficient.	Barometer.	Elevation above sea level.	Coefficient.	Temperature (Fahr.).	Coefficient.
Ins.	*Feet.*		*Ins.*	*Feet.*		°	
30.5	−451	1.03	25.4	4,535	0.86	−24	1.17
30.2	−181	1.02	25.1	4,859	.85	17	1.15
30.0	00	1.01	24.8	5,186	.84	− 9	1.13
29.9	+ 91	1.01	24.5	5,518	.83	0	1.11
29.6	366	1.00	24.2	5,854	.82	+ 8	1.09
29.3	643	.99	23.9	6,194	.81	16	1.07
29.0	924	.98	23.6	6,538	.80	25	1.05
28.7	1,207	.97	23.3	6,887	.79	35	1.03
28.4	1,493	.96	23.0	7,239	.78	40	1.02
28.1	1,783	.95	22.7	7,597	.77	45	1.01
27.8	2,075	.94	22.4	7,960	.76	50	1.00
27.5	2,371	.93	22.1	8,327	.75	55	.99
27.2	2,670	.92	21.8	8,700	.74	60	.98
26.9	2,972	.91	21.5	9,077	.73	66	.97
26.6	3,277	.90	21.2	9,460	.72	77	.95
26.3	3,586	.89	20.9	9,848	.71	88	.93
26.0	3,899	.88	20.6	10,242	.70	100	.91
25.7	4,215	.87	20.3	10,642	.69	114	.89
25.4	4,535	.86	20.0	11,047	.68	128	.87

Any true refraction either in zenith or polar distance=tabulated refraction × coefficient for barometric pressure × coefficient for temperature.

The differences between the true and the tabulated refractions are generally small and generally negligible excepting for the combined effect of low apparent altitude of observation with high elevation above sea level or extremes of temperature.

See Smithsonian Meteorological Tables for revised data.

896504°—56——14

TABLE 22.—COEFFICIENTS FOR COMPUTING ERRORS IN AZIMUTH DUE TO SMALL ERRORS IN DECLINATION OR LATITUDE, TABULATED FOR 1′ ERRORS.

Lat.		Hours from noon. 2	3	4	5	6
°						
25	Decl.	2.21	1.56	1.27	1.14	1.10
	Lat.	1.91	1.10	0.64	0.30	0.00
30	Decl.	2.31	1.62	1.33	1.20	1.15
	Lat.	2.00	1.15	0.67	0.31	0.00
35	Decl.	2.44	1.73	1.41	1.26	1.22
	Lat.	2.11	1.22	0.70	0.33	0.00
40	Decl.	2.61	1.85	1.51	1.35	1.31
	Lat.	2.26	1.31	0.75	0.35	0.00
45	Decl.	2.83	2.00	1.63	1.46	1.41
	Lat.	2.45	1.41	0.82	0.38	0.00
50	Decl.	3.11	2.20	1.80	1.61	1.56
	Lat.	2.69	1.56	0.90	0.42	0.00
55	Decl.	3.49	2.47	2.01	1.80	1.74
	Lat.	3.02	1.74	1.01	0.47	0.00
60	Decl.	4.00	2.83	2.31	2.07	2.00
	Lat.	3.46	2.00	1.15	0.54	0.00
65	Decl.	4.73	3.35	2.73	2.45	2.37
	Lat.	4.10	2.37	1.37	0.63	0.00
70	Decl.	5.85	4.13	3.38	3.03	2.92
	Lat.	5.06	2.92	1.69	0.78	0.00

The coefficients for noon become infinitely large.

Direction of errors in azimuth from true south:

To the east in the a. m., to the west in the p. m.—
- Latitude set or assumed to the N. of the true latitude, noon to six hours.
- Latitude set or assumed to the S. of the true latitude, beyond six hours.
- Declination set or assumed to the S. of the true declination, all hours.

To the west in the a. m., to the east in the p. m.—
- Latitude set or assumed to the S. of the true latitude, noon to six hours.
- Latitude set or assumed to the N. of the true latitude, beyond six hours.
- Declination set or assumed to the N. of the true declination, all hours.

The error is considered as the erroneous fixation of the south point.

TABLE 23.—MEAN REFRACTIONS IN POLAR DISTANCE.

Bar.=29.6 ins. Temp.=50° F.

Hours from noon.	Latitude. 25° 0′	27° 30′	30° 0′	25° 0′	27° 30′	30° 0′	25° 0′	27° 30′	30° 0′
	Decl.=23° 27′ N.			Decl.=22° 30′ N.			Decl.=20° 0′ N.		
	′ ″	′ ″	″	′ ″	′ ″	″	′ ″	′ ″	″
0	0 2	0 4	0 7	0 3	0 5	0 8	0 5	0 8	0 10
2	0 5	0 7	0 10	0 6	0 8	0 11	0 9	0 11	0 14
3	0 8	0 12	0 16	0 10	0 13	0 17	0 14	0 17	0 20
4	0 20	0 23	0 27	0 22	0 25	0 29	0 25	0 28	0 32
5	0 44	0 49	0 53	0 46	0 51	0 55	0 50	0 55	1 0
6	2 7	2 9	2 10	2 12	2 14	2 16	2 29	2 31	2 33
	Decl.=17° 30′ N.			Decl.=15° 0′ N.			Decl.=12° 30′ N.		
0	0 8	0 10	0 13	0 10	0 13	0 16	0 13	0 16	0 18
2	0 11	0 13	0 16	0 14	0 16	0 19	0 16	0 19	0 22
3	0 16	0 19	0 23	0 19	0 22	0 26	0 22	0 25	0 29
4	0 28	0 31	0 35	0 31	0 35	0 39	0 34	0 38	0 43
5	0 54	1 0	1 5	0 59	1 5	1 10	1 4	1 11	1 17
6	2 49	2 52	2 55	3 15	3 19	3 22	3 49	3 53	3 57
	Decl.=10° 0′ N.			Decl.=7° 30′ N.			Decl.=5° 0′ N.		
0	0 16	0 18	0 21	0 18	0 21	0 24	0 21	0 24	0 27
2	0 19	0 22	0 25	0 22	0 25	0 28	0 25	0 28	0 31
3	0 25	0 28	0 32	0 28	0 31	0 35	0 31	0 34	0 38
4	0 37	0 42	0 47	0 41	0 46	0 51	0 46	0 51	0 56
5	1 10	1 17	1 24	1 17	1 25	1 33	1 25	1 33	1 42
	Decl.=2° 30′ N.			Decl.=0° 0′.			Decl.=2° 30′ S.		
0	0 24	0 27	0 30	0 27	0 30	0 33	0 30	0 33	0 37
2	0 28	0 31	0 34	0 31	0 34	0 38	0 34	0 38	0 42
3	0 34	0 38	0 42	0 38	0 42	0 47	0 41	0 46	0 51
4	0 50	0 55	1 1	0 55	1 0	1 6	0 59	1 5	1 12
5	1 33	1 43	1 53	1 42	1 53	2 5	1 52	2 5	2 20
	Decl.=5° 0′ S.			Decl.=7° 30′ S.			Decl.=10° 0′ S.		
0	0 33	0 37	0 40	0 37	0 40	0 44	0 40	0 44	0 48
2	0 38	0 42	0 47	0 42	0 46	0 51	0 46	0 50	0 55
3	0 45	0 50	0 56	0 49	0 55	1 1	0 53	1 0	1 7
4	1 4	1 11	1 19	1 10	1 18	1 27	1 17	1 26	1 36
5	2 5	2 23	2 41	2 20	2 43	3 6	2 39	3 5	3 36
	Decl.=12° 30′ S.			Decl.=15° 0′ S.			Decl.=17° 30′ S.		
0	0 44	0 48	0 53	0 48	0 53	0 58	0 53	0 58	1 3
2	0 50	0 55	1 0	0 54	1 0	1 6	0 59	1 5	1 12
3	0 58	1 5	1 13	1 4	1 12	1 20	1 11	1 19	1 28
4	1 24	1 34	1 46	1 31	1 43	1 57	1 40	1 54	2 11
4½	1 51	2 9	2 28	2 2	2 23	2 47	2 14	2 40	3 13
	Decl.=20° 0′ S.			Decl.=22° 30′ S.			Decl.=23° 27′ S.		
0	0 58	1 3	1 9	1 3	1 9	1 15	1 5	1 12	1 18
2	1 5	1 11	1 18	1 11	1 18	1 26	1 13	1 20	1 29
3	1 18	1 27	1 37	1 25	1 35	1 46	1 28	1 38	1 50
4	1 52	2 8	2 28	2 4	2 23	2 48	2 10	2 32	2 58
4½	2 30	3 3	3 47	2 53	3 34	4 31	3 11	3 57	5 0

TABLE 23.—MEAN REFRACTIONS IN POLAR DISTANCE.

Bar.=29.6 ins. Temp.=50° F.

Hours from noon.	Latitude. 30° 0′	32° 30′	35° 0′	37° 30′	40° 0′	42° 30′	45° 0′	47° 30′	50° 0′
	Declination of the sun, 23° 27′ N.								
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0 7	0 9	0 12	0 15	0 18	0 20	0 23	0 26	0 29
2	0 10	0 13	0 16	0 19	0 22	0 24	0 27	0 30	0 34
3	0 16	0 19	0 22	0 25	0 28	0 31	0 35	0 38	0 41
4	0 27	0 30	0 34	0 37	0 41	0 45	0 49	0 53	0 56
5	0 53	0 57	1 0	1 4	1 7	1 11	1 14	1 17	1 20
6	2 10	2 11	2 11	2 11	2 11	2 11	2 11	2 11	2 12
6½	4 35	4 14	3 57	3 44	3 33	3 23	3 15	3 7	3 0
	Declination of the sun, 22° 30′ N.								
0	0 8	0 10	0 13	0 16	0 18	0 21	0 24	0 27	0 30
2	0 11	0 14	0 17	0 20	0 23	0 26	0 29	0 32	0 35
3	0 17	0 20	0 23	0 26	0 29	0 32	0 36	0 40	0 43
4	0 29	0 32	0 35	0 39	0 43	0 47	0 51	0 55	0 58
5	0 55	0 59	1 2	1 6	1 10	1 13	1 16	1 20	1 23
6	2 16	2 17	2 17	2 17	2 17	2 17	2 17	2 17	2 18
6½	5 3	4 38	4 17	4 1	3 47	3 34	3 23	3 14	3 7
	Declination of the sun, 20° 0′ N.								
0	0 10	0 13	0 16	0 18	0 21	0 24	0 27	0 30	0 33
2	0 14	0 16	0 19	0 22	0 25	0 28	0 32	0 35	0 38
3	0 20	0 23	0 26	0 29	0 32	0 35	0 39	0 43	0 48
4	0 32	0 35	0 39	0 43	0 47	0 51	0 55	0 59	1 2
5	1 0	1 4	1 8	1 12	1 16	1 20	1 24	1 28	1 31
5½	1 27	1 32	1 37	1 41	1 45	1 49	1 53	1 56	1 58
6	2 33	2 34	2 34	2 35	2 35	2 36	2 36	2 36	2 36
6½	6 12	5 40	5 14	4 52	4 34	4 19	4 7	3 57	3 49
	Declination of the sun, 17° 30′ N.								
0	0 13	0 16	0 18	0 21	0 24	0 27	0 30	0 33	0 37
2	0 16	0 19	0 22	0 25	0 28	0 31	0 35	0 38	0 42
3	0 23	0 26	0 29	0 32	0 36	0 39	0 43	0 47	0 52
4	0 35	0 39	0 43	0 47	0 51	0 55	1 0	1 4	1 7
5	1 5	1 10	1 14	1 19	1 23	1 27	1 32	1 37	1 40
5½	1 37	1 42	1 46	1 51	1 55	2 0	2 4	2 7	2 10
6	2 55	2 56	2 56	2 57	2 58	2 59	2 59	2 59	2 59
	Declination of the sun, 15° 0′ N.								
0	0 16	0 18	0 21	0 24	0 27	0 30	0 33	0 37	0 40
2	0 19	0 22	0 25	0 28	0 31	0 34	0 38	0 42	0 47
3	0 26	0 29	0 33	0 36	0 40	0 44	0 48	0 52	0 57
4	0 39	0 44	0 48	0 52	0 56	1 0	1 5	1 9	1 14
5	1 10	1 16	1 21	1 26	1 31	1 36	1 41	1 46	1 51
5½	1 48	1 53	1 58	2 3	2 8	2 13	2 18	2 22	2 26
6	3 22	3 24	3 25	3 26	3 27	3 28	3 28	3 29	3 30

TABLE 23.—MEAN REFRACTIONS IN POLAR DISTANCE.

Bar.=29.6 ins Temp.=50° F.

Hours from noon.	Latitude. 30° 0′	32° 30′	35° 0′	37° 30′	40° 0′	42° 30′	45° 0′	47° 30′	50° 0′
	Declination of the sun, 12° 30′ N.								
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0 18	0 21	0 24	0 27	0 30	0 33	0 37	0 40	0 44
2	0 22	0 25	0 28	0 31	0 34	0 38	0 42	0 46	0 51
3	0 29	0 32	0 36	0 40	0 44	0 48	0 52	0 57	1 2
4	0 43	0 47	0 52	0 56	1 1	1 6	1 11	1 16	1 21
5	1 17	1 23	1 29	1 35	1 40	1 46	1 51	1 57	2 4
5½	2 0	2 6	2 12	2 18	2 24	2 30	2 36	2 41	2 46
6	3 57	4 0	4 4	4 6	4 7	4 8	4 8	4 10	4 11
	Declination of the sun, 10° 0′ N.								
0	0 21	0 24	0 27	0 30	0 33	0 37	0 40	0 44	0 48
2	0 25	0 28	0 32	0 35	0 38	0 42	0 47	0 51	0 56
3	0 32	0 35	0 39	0 44	0 49	0 53	0 57	1 2	1 7
4	0 47	0 51	0 56	1 1	1 7	1 12	1 18	1 23	1 29
5	1 24	1 31	1 38	1 45	1 51	1 57	2 4	2 11	2 19
5½	2 14	2 22	2 30	2 38	2 45	2 52	2 59	3 6	3 13
6	4 50	4 54	4 58	5 1	5 3	5 5	5 6	5 8	5 9
	Declination of the sun, 7° 30′ N.								
0	0 24	0 27	0 30	0 33	0 37	0 40	0 44	0 48	0 53
2	0 28	0 31	0 35	0 38	0 42	0 46	0 51	0 56	1 1
3	0 35	0 39	0 43	0 48	0 53	0 58	1 2	1 8	1 13
4	0 51	0 56	1 1	1 7	1 13	1 19	1 25	1 32	1 38
5	1 33	1 40	1 48	1 56	2 4	2 12	2 20	2 29	2 38
5½	2 31	2 41	2 52	3 2	3 11	3 20	3 29	3 39	3 48
	Declination of the sun, 5° 0′ N.								
0	0 27	0 30	0 33	0 37	0 40	0 44	0 48	0 53	0 58
2	0 31	0 34	0 38	0 42	0 47	0 51	0 56	1 1	1 6
3	0 38	0 43	0 48	0 53	0 58	1 3	1 8	1 14	1 20
4	0 56	1 1	1 7	1 13	1 19	1 26	1 33	1 41	1 49
5	1 42	1 51	2 0	2 10	2 20	2 30	2 40	2 51	3 2
5½	2 53	3 5	3 18	3 31	3 44	3 57	4 10	4 23	4 35
	Declination of the sun, 2° 30′ N.								
0	0 30	0 33	0 37	0 40	0 44	0 48	0 53	0 58	1 3
2	0 34	0 38	0 42	0 46	0 51	0 56	1 1	1 6	1 12
3	0 42	0 47	0 52	0 58	1 3	1 9	1 14	1 21	1 28
4	1 1	1 7	1 14	1 20	1 27	1 35	1 43	1 52	2 1
5	1 53	2 3	2 14	2 27	2 39	2 52	3 5	3 19	3 33
5½	3 21	3 37	3 53	4 11	4 29	4 49	5 9	5 28	5 47

TABLE 23.—MEAN REFRACTIONS IN POLAR DISTANCE.

Bar.=29.6 ins. Temp.=50° F.

Hours from noon.	Latitude. 30° 0′	32° 30′	35° 0′	37° 30′	40° 0′	42° 30′	45° 0′	47° 30′	50° 0′
	Declination of the sun, 0° 0′.								
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0 33	0 37	0 40	0 44	0 48	0 53	0 58	1 3	1 9
2	0 38	0 42	0 47	0 51	0 56	1 1	1 7	1 12	1 18
3	0 47	0 52	0 57	1 3	1 9	1 15	1 21	1 28	1 36
4	1 6	1 13	1 21	1 28	1 36	1 45	1 54	2 4	2 16
4½	1 26	1 35	1 44	1 54	2 4	2 16	2 29	2 42	2 55
5	2 5	2 18	2 32	2 47	3 2	3 19	3 37	3 56	4 15
	Declination of the sun, 2° 30′ S.								
0	0 37	0 40	0 44	0 48	0 53	0 58	1 3	1 9	1 15
2	0 42	0 46	0 51	0 56	1 1	1 7	1 13	1 20	1 27
3	0 51	0 57	1 2	1 8	1 15	1 22	1 29	1 37	1 46
4	1 12	1 20	1 28	1 37	1 46	1 57	2 8	2 20	2 34
4½	1 34	1 45	1 56	2 8	2 21	2 34	2 50	3 7	3 25
5	2 20	2 36	2 54	3 13	3 32	3 55	4 20	4 47	5 16
	Declination of the sun, 5° 0′ S.								
0	0 40	0 44	0 48	0 53	0 58	1 3	1 9	1 15	1 22
2	0 47	0 51	0 56	1 1	1 7	1 13	1 20	1 28	1 36
3	0 56	1 2	1 8	1 14	1 21	1 29	1 38	1 48	1 59
4	1 19	1 28	1 37	1 47	1 58	2 11	2 25	2 40	2 57
4½	1 44	1 57	2 10	2 24	2 40	2 57	3 17	3 40	4 6
5	2 41	2 59	3 22	3 47	4 15	4 46	5 21	6 2	6 52
	Declination of the sun, 7° 30′ S.								
0	0 44	0 48	0 53	0 58	1 3	1 9	1 15	1 22	1 30
2	0 51	0 55	1 1	1 6	1 13	1 20	1 28	1 37	1 46
3	1 1	1 8	1 15	1 22	1 30	1 38	1 48	2 0	2 14
4	1 27	1 37	1 47	1 59	2 12	2 28	2 45	3 4	3 27
4½	1 56	2 11	2 27	2 44	3 4	3 27	3 53	4 26	5 4
5	3 6	3 30	3 59	4 34	5 13	5 57	6 45		
	Declination of the sun, 10° 0′ S.								
0	0 48	0 53	0 58	1 3	1 9	1 15	1 22	1 30	1 40
2	0 55	1 0	1 6	1 12	1 19	1 27	1 36	1 46	1 58
3	1 7	1 14	1 22	1 30	1 39	1 49	2 1	2 15	2 31
4	1 36	1 47	1 59	2 13	2 30	2 49	3 10	3 36	4 7
4½	2 11	2 28	2 47	3 10	3 36	4 8	4 44	5 30	6 28

TABLE 23.—MEAN REFRACTIONS IN POLAR DISTANCE.

Bar.=29.6 ins. Temp.=50° F.

Hours from noon.	Latitude. 30°0′	32°30′	35°0′	37°30′	40°0′	42°30′	45°0′	47°30′	50°0′
	Declination of the sun, 12° 30′S.								
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0 53	0 58	1 3	1 9	1 15	1 22	1 30	1 40	1 50
2	1 0	1 6	1 13	1 20	1 27	1 36	1 46	1 57	2 11
3	1 13	1 21	1 30	1 39	1 50	2 1	2 16	2 33	2 52
4	1 46	1 59	2 14	2 31	2 52	3 16	3 43	4 19	5 4
4½	2 28	2 49	3 13	3 44	4 20	5 5	6 0		
	Declination of the sun, 15° 0′ S.								
0	0 58	1 3	1 9	1 15	1 22	1 30	1 40	1 50	2 3
2	1 6	1 12	1 20	1 28	1 36	1 46	1 58	2 10	2 26
3	1 20	1 28	1 38	1 49	2 2	2 16	2 34	2 54	3 20
3½	1 34	1 45	1 57	2 12	2 29	2 48	3 12	3 42	4 21
4	1 57	2 14	2 33	2 55	3 20	3 52	4 30	5 18	6 32
4½	2 47	3 15	3 47	4 29	5 20	6 28			
	Declination of the sun, 17° 30′ S.								
0	1 3	1 9	1 15	1 22	1 30	1 40	1 50	2 3	2 18
2	1 12	1 19	1 28	1 37	1 46	1 58	2 12	2 26	2 48
3	1 28	1 37	1 48	2 1	2 17	2 35	2 57	3 22	4 0
3½	1 43	1 57	2 12	2 29	2 50	3 15	3 47	4 28	5 24
4	2 11	2 31	2 55	3 23	3 57	4 42	5 40	6 56	
	Declination of the sun, 20° 0′ S.								
0	1 9	1 15	1 22	1 30	1 40	1 50	2 3	2 18	2 37
2	1 18	1 27	1 36	1 46	1 58	2 12	2 29	2 47	3 17
3	1 37	1 47	2 0	2 16	2 35	2 58	3 26	4 0	4 54
3½	1 54	2 10	2 29	2 50	3 17	3 51	4 36	5 30	7 7
4	2 28	2 52	3 21	3 56	4 47	5 58			
	Declination of the sun, 22° 30′ S.								
0	1 15	1 22	1 30	1 40	1 50	2 3	2 18	2 37	3 1
2	1 26	1 35	1 45	1 57	2 12	2 29	2 49	3 13	3 53
2½	1 34	1 45	1 57	2 11	2 28	2 49	3 16	3 50	4 32
3	1 46	1 59	2 15	2 34	2 57	3 26	4 4	4 56	6 10
3½	2 7	2 25	2 48	3 18	3 51	4 38	5 43		
4	2 48	3 19	3 57	4 46	5 56				
	Declination of the sun, 23° 27′ S.								
0	1 18	1 25	1 34	1 44	1 55	2 9	2 25	2 46	3 12
2	1 29	1 39	1 50	2 3	2 18	2 37	3 0	3 28	4 10
2½	1 36	1 49	2 3	2 18	2 37	3 1	3 31	4 11	5 9
3	1 50	2 6	2 23	2 44	3 9	3 43	4 27	5 25	7 1
3½	2 15	2 33	2 58	3 32	4 12	5 4	6 25		
4	2 58	3 32	4 17	5 14	6 45				

TABLE 23.—MEAN REFRACTIONS IN POLAR DISTANCE.

Bar.=29.6 ins. Temp.=50° F.

Hours from noon.	Latitude. 50° 0′	52° 30′	55° 0′	57° 30′	60° 0′	62° 30′	65° 0′	67° 30′	70° 0′
	Declination of the sun, 23° 27′ N.								
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0 29	0 32	0 35	0 39	0 42	0 46	0 51	0 56	1 1
2	0 34	0 37	0 41	0 44	0 48	0 52	0 57	1 1	1 6
3	0 41	0 44	0 48	0 52	0 56	1 0	1 5	1 9	1 14
4	0 56	0 59	1 2	1 5	1 9	1 13	1 18	1 22	1 26
5	1 20	1 23	1 26	1 29	1 32	1 35	1 38	1 41	1 44
6	2 12	2 12	2 12	2 12	2 12	2 12	2 12	2 12	2 12
	Declination of the sun, 22° 30′ N.								
0	0 30	0 33	0 37	0 40	0 44	0 48	0 53	0 58	1 3
2	0 35	0 38	0 42	0 45	0 49	0 53	0 58	1 3	1 8
3	0 43	0 46	0 50	0 54	0 58	1 2	1 7	1 11	1 16
4	0 58	1 1	1 4	1 7	1 11	1 15	1 20	1 24	1 29
5	1 23	1 26	1 29	1 32	1 35	1 38	1 42	1 45	1 48
6	2 18	2 18	2 18	2 18	2 18	2 18	2 18	2 18	2 18
	Declination of the sun, 20° 0′ N.								
0	0 33	0 37	0 40	0 44	0 48	0 53	0 58	1 3	1 9
2	0 38	0 42	0 46	0 50	0 54	0 58	1 3	1 9	1 15
3	0 48	0 51	0 55	0 59	1 3	1 8	1 13	1 18	1 24
4	1 2	1 6	1 10	1 14	1 18	1 22	1 27	1 32	1 38
5	1 31	1 34	1 38	1 41	1 45	1 49	1 53	1 57	2 1
6	2 36	2 36	2 36	2 36	2 36	2 36	2 36	2 37	2 37
	Declination of the sun, 17° 30′ N.								
0	0 37	0 40	0 44	0 48	0 53	0 58	1 3	1 9	1 15
2	0 42	0 46	0 50	0 54	0 59	1 4	1 9	1 15	1 22
3	0 52	0 56	1 0	1 4	1 9	1 14	1 20	1 26	1 33
4	1 7	1 12	1 17	1 21	1 26	1 31	1 36	1 42	1 49
5	1 40	1 44	1 48	1 52	1 57	2 2	2 7	2 11	2 15
6	2 59	2 59	2 59	2 59	2 59	2 59	2 59	3 0	3 0
	Declination of the sun, 15° 0′ N.								
0	0 40	0 44	0 48	0 53	0 58	1 3	1 9	1 15	1 22
2	0 47	0 51	0 55	1 0	1 5	1 10	1 16	1 23	1 30
3	0 57	1 1	1 6	1 10	1 15	1 21	1 28	1 35	1 43
4	1 14	1 19	1 24	1 29	1 35	1 41	1 47	1 54	2 1
5	1 51	1 56	2 1	2 6	2 12	2 17	2 23	2 28	2 34
6	3 30	3 30	3 30	3 30	3 30	3 30	3 30	3 31	3 31

TABLE 23.—MEAN REFRACTIONS IN POLAR DISTANCE.

Bar.=29.6 ins. Temp.=50° F.

Hours from noon.	Latitude. 50° 0′	52° 30′	55° 0′	57° 30′	60° 0′	62° 30′	65° 0′	67° 30′	70° 0′
	Declination of the sun, 12° 30′ N.								
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0 44	0 48	0 53	0 58	1 3	1 9	1 15	1 22	1 30
2	0 51	0 56	1 1	1 6	1 11	1 17	1 23	1 30	1 39
3	1 2	1 7	1 12	1 17	1 22	1 29	1 37	1 45	1 54
4	1 21	1 27	1 32	1 38	1 45	1 52	1 59	2 7	2 16
5	2 4	2 10	2 16	2 22	2 29	2 35	2 42	2 49	2 57
6	4 11	4 12	4 12	4 12	4 12	4 12	4 13	4 13	4 13
	Declination of the sun, 10° 0′ N.								
0	0 48	0 53	0 58	1 3	1 9	1 15	1 22	1 30	1 40
2	0 56	1 1	1 7	1 12	1 18	1 25	1 32	1 41	1 50
3	1 7	1 12	1 18	1 24	1 31	1 39	1 47	1 56	2 6
4	1 29	1 35	1 42	1 49	1 57	2 5	2 14	2 24	2 34
5	2 19	2 26	2 34	2 42	2 50	2 58	3 7	3 16	3 26
6	5 9	5 9	5 10	5 10	5 10	5 11	5 11	5 12	5 12
	Declination of the sun, 7° 30′ N.								
0	0 53	0 58	1 3	1 9	1 15	1 22	1 30	1 40	1 50
2	1 1	1 7	1 12	1 19	1 26	1 34	1 42	1 52	2 3
3	1 13	1 19	1 25	1 33	1 41	1 50	1 59	2 9	2 21
4	1 38	1 45	1 53	2 2	2 11	2 21	2 32	2 44	2 57
5	2 38	2 47	2 57	3 7	3 17	3 28	3 40	3 53	4 6
	Declination of the sun, 5° 0′ N.								
0	0 58	1 3	1 9	1 15	1 22	1 30	1 40	1 50	2 3
2	1 6	1 12	1 18	1 26	1 35	1 44	1 53	2 4	2 18
3	1 20	1 27	1 34	1 43	1 52	2 2	2 14	2 27	2 40
4	1 49	1 57	2 5	2 16	2 28	2 41	2 55	3 10	3 27
5	3 2	3 14	3 26	3 39	3 53	4 8	4 24	4 42	5 1
	Declination of the sun, 2° 30′ N.								
0	1 3	1 9	1 15	1 22	1 30	1 40	1 50	2 3	2 18
2	1 12	1 19	1 25	1 34	1 44	1 55	2 6	2 20	2 37
3	1 28	1 35	1 44	1 54	2 5	2 18	2 32	2 48	3 5
4	2 1	2 12	2 21	2 34	2 49	3 6	3 24	3 44	4 8
5	3 33	3 50	4 6	4 25	4 45	5 7	5 32	6 0	6 31
	Declination of the sun, 0° 0′.								
0	1 9	1 15	1 22	1 30	1 40	1 50	2 3	2 18	2 37
2	1 18	1 26	1 34	1 43	1 54	2 6	2 20	2 38	3 1
3	1 36	1 45	1 55	2 7	2 20	2 35	2 53	3 14	3 39
4	2 16	2 29	2 43	2 58	3 16	3 36	4 0	4 28	5 3
5	4 15	4 38	5 2	5 29	5 59	6 33	7 13	7 59	8 53

TABLE 24.—TRIGONOMETRIC FORMULAS FOR THE SOLUTION OF PLANE TRIANGLES.

Let A = angle BAC = arc BF, and let the radius $AF = AB = AH = 1$.
We then have

$\sin A = BC$
$\cos A = AC$
$\tan A = DF$
$\cot A = HG$
$\sec A = AD$
$\operatorname{cosec} A = AG$
$\operatorname{versin} A = CF = BE$
$\operatorname{covers} A = BK = HL$
$\operatorname{exsec} A = BD$
$\operatorname{coexsec} A = FG$
$\operatorname{chord} A = BF$
$\operatorname{chord} 2A = BI = 2BC$

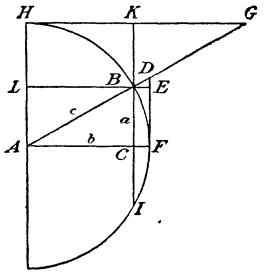

FIG 1.

In the right-angled triangle ABC
Let $AB = c$, $AC = b$, and $BC = a$.
We then have:

1. $\sin A = \frac{a}{c} = \cos B$
2. $\cos A = \frac{b}{c} = \sin B$
3. $\tan A = \frac{a}{b} = \cot B$
4. $\cot A = \frac{b}{a} = \tan B$
5. $\sec A = \frac{c}{b} = \operatorname{cosec} B$
6. $\operatorname{cosec} A = \frac{c}{a} = \sec B$
7. $\operatorname{vers} A = \frac{c-b}{c} = \operatorname{covers} B$
8. $\operatorname{exsec} A = \frac{c-b}{b} = \operatorname{coexsec} B$
9. $\operatorname{covers} A = \frac{c-a}{c} = \operatorname{versin} B$
10. $\operatorname{coexsec} A = \frac{c-a}{a} = \operatorname{exsec} B$
11. $a = c \sin A = b \tan A$
12. $b = c \cos A = a \cot A$
13. $c = \frac{a}{\sin A} = \frac{b}{\cos A}$
14. $a = c \cos B = b \cot B$
15. $b = c \sin B = a \tan B$
16. $c = \frac{a}{\cos B} = \frac{b}{\sin B}$
17. $a = \sqrt{(c+b)(c-b)}$
18. $b = \sqrt{(c+a)(c-a)}$
19. $c = \sqrt{a^2+b^2}$
20. $C = 90° = A+B$

21. $\text{area} = \frac{ab}{2}$

TABLE 24.—TRIGONOMETRIC FORMULAS FOR THE SOLUTION OF PLANE TRIANGLES.

SOLUTION OF OBLIQUE TRIANGLES.

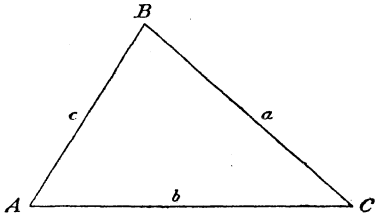

FIG. 2.

	GIVEN.	SOUGHT.	FORMULA.
22	A, R	C, b, c	$C = 180° - (A+B)$, $b = \frac{a}{\sin A} \cdot \sin B$, $c = \frac{a}{\sin A} \sin (A+B)$
23	A, a, b	B, C, c	$\sin B = \frac{\sin A}{a} \cdot b$, $C = 180° - (A+B)$, $c = \frac{a}{\sin A} \cdot \sin C$.
24	C, a, b	$\frac{1}{2}(A+B)$	$\frac{1}{2}(A+B) = 90° - \frac{1}{2}C$
25		$\frac{1}{2}(A-B)$	$\tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A+B)$
26		A, B	$A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B)$, $B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B)$
27		c	$c = (a+b)\frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} = \sqrt{a^2+b^2-2ab\cos C}$
28		area	$\text{area} = \frac{1}{2} ab \sin C$.
29	a, b, c	A	Let $s = \frac{1}{2}(a+b+c)$; $\sin \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}$
30			$\cos \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$; $\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
31			$\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$; $\cos A = \frac{b^2+c^2-a^2}{2bc}$
32		area	$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$
33	A, B, C, a	area	$\text{area} = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$

TABLE 25.—TRIGONOMETRIC FORMULAS FOR THE SOLUTION OF STADIA MEASUREMENTS, OBSERVATIONS FOR TIME, LATITUDE AND AZIMUTH, AND PROBLEMS IN CONVERGENCY.

NOTATION.

"THE EPHEMERIS."

"Ephemeris of the Sun, Polaris and Other Selected Stars," a supplement to the Manual of Surveying Instructions.

"$\neq$": The symbol for approximation.

"v."

Observed vertical angle; in altitude observations on the sun, the reductions to the sun's center both vertically and horizontally, as well as instrumental errors, are eliminated by taking direct and reversed observations on the opposite limbs of the sun, and the mean observed vertical angle to the sun's center is to be considered "v" in the notation. In single observations the vertical reduction to the sun's center = 16′; a refinement is had by referring to the Ephemeris for the value of the sun's semidiameter for the date of observation.

"h."

True vertical angle to the sun's center, or to Polaris, in altitude observations, after correction for refraction: $h = v -$ refraction in zenith distance; a refinement is had in altitude observations on the sun by adding the value of the sun's parallax = 8″.9 cos v, opposite in effect to refraction.

"ζ."

Zeta: True zenith distance of the sun's center.

$$\zeta = 90° - h.$$

"ϕ."

Phi: Latitude of the station of observation.

"λ."

Lambda: Longitude of the station of observation.

"δ."

Delta: Declination of the sun or Polaris; to be taken from the Ephemeris for the date of observation; the declination of the sun is to be corrected in hourly difference to the longitude of the station and to the time of observation: north declinations are treated as positive and south declinations as negative; a northerly hourly motion is treated as positive and a southerly hourly motion is treated as negative; in the use of the solar attachment the declination of the sun is to be corrected for refraction in polar distance, always north.

"A."

Azimuth angle from the true meridian to Polaris, or to the sun's center; in the following analytical solutions "A" is referred to the north point unless otherwise modified, and the reductions are symmetrical either east or west of the meridian; all determinations for azimuth imply the recording of horizontal angles from a fixed reference point to Polaris or to the sun, or a point marked on the ground to define the direction of observation, the mean horizontal angle, or the mean point in direction, being used in the determination; in single observations on the sun, the reduction to the sun's center in azimuth $= \frac{16'}{\cos v}$; a refinement in the value of the sun's semidiameter is had by referring to the Ephemeris for the date of observation.

SIGNS OF TRIGONOMETRIC FUNCTIONS.

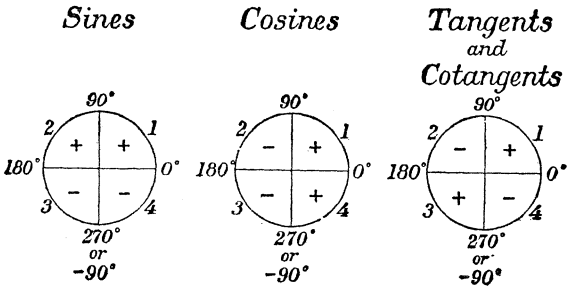

FIG. 3.

STADIA MEASUREMENTS.

Hor. dis.: The true horizontal distance from the center of the instrument to the rod.

Diff. elev.: The true vertical distance from the height of the instrument to the center point between the two targets of the rod.

"r": Vertical rod reading.

"K": The wire interval or ratio, to be determined in the field by frequent tests under working conditions in comparison with steel tape measurement, solving the formula given below for "K," hor. dis. known.

"c": Distance from the center of the instrument to the object glass.

"f": Distance from the plane of the cross-wires to the object glass.

$$\text{Hor. dis.} = K r \cos^2 v + (c + f) \cos v.$$
$$\text{Diff. elev.} = K r \tfrac{1}{2} \sin 2v + (c + f) \sin v.$$

In public land surveying it is convenient to have fixed stadia wires with a ratio of 1:132 so that the sum of two rod readings in feet will be equivalent to a ratio of 1:66, or a reduced distance in chains; it is also convenient to reduce the error in the wire interval to the error in 10 chs., and to eliminate the error by applying to the reduced distance the proper amount taken from the table of proportional parts. With a ratio of 1:100, using a rod graduated to links, the elimination of the error in the wire interval is conveniently made in the same manner. With a ratio of 1:100, using a rod graduated to feet, the reduction is simplified by determining the logarithm of the true "K," rod in feet and horizontal distance in chains units, accomplishing the reduction of $K r \cos^2 v$ by logarithmic functions.

TIME.

Conversion of standard time into local mean time.—Watch reading ± watch error in standard time by comparison ± correction for longitude; the correction for longitude is additive east and subtractive west of the standard meridian of the time belt; the conversion table "degrees to time" is convenient in this reduction.

Conversion of apparent time into local mean time.—Apparent time of observation ± the equation of time; the equation of time is to be taken from the Ephemeris for the date of observation and corrected for the longitude and time of observation, conveniently interpolated as the interval from Greenwich noon to the time of observation;

the watch error in local mean time is then found by taking the difference between the watch reading at the epoch of the observation and the reduced local mean time of observation.

Local mean time of upper culmination of Polaris.—The Greenwich mean time of upper culmination of Polaris is to be taken from the Ephemeris for the date of observation; the amount to be subtracted from the Greenwich mean time of upper culmination of Polaris to obtain the local mean time of upper culmination of Polaris, in which the argument is the longitude west from Greenwich, is obtained from the table of sidereal conversions without computation.

Local mean time of elongation of Polaris.—The mean time of elongation of Polaris, Greenwich meridian, latitude 40°, is to be taken from the Ephemeris for the date of observation; the amount to be subtracted from the mean time of elongation of Polaris, Greenwich meridian, latitude 40°, to obtain the mean time of elongation of Polaris, local meridian, latitude 40°, in which the argument is the longitude west from Greenwich, is obtained from the table of sidereal conversions without computation. The amount to apply to the local mean time of elongation of Polaris latitude 40° to obtain the local mean time of elongation of Polaris latitude of observation is tabulated in the Ephemeris in connection with the table of azimuths of Polaris at elongation.

Conversion of a mean time interval into a sidereal time interval, or vice versa.—The amount to apply to one time interval to obtain the other time interval is obtained from the table of sidereal conversions without computation.

Hour angles of Polaris.—A mean time hour angle of Polaris west of the meridian is the mean time interval from the local mean time of upper culmination of Polaris to the local mean time of observation of Polaris; a mean time hour angle of Polaris east of the meridian is the mean time interval from the local mean time of observation of Polaris to the local mean time of upper culmination of Polaris.

Mean time hour angle of Polaris at elongation.—"t"=the sidereal hour angle in angular measure, this converted into time measure, and this in turn converted from a sidereal time interval into a mean time interval gives the mean time hour angle of Polaris at elongation:

$$\text{Cos } t = \text{cotan } \delta \text{ tan } \phi$$

Altitude observation of the sun for apparent time.—"t"=hour angle from apparent noon in angular measure; reverse the signs of "δ" for south declinations:

$$\text{Tan } \tfrac{1}{2} t = \sqrt{\frac{\sin \frac{1}{2}(\zeta+\phi-\delta)\sin \frac{1}{2}(\zeta-\phi+\delta)}{\cos \frac{1}{2}(\zeta+\phi+\delta)\cos \frac{1}{2}(\zeta-\phi-\delta)}}$$

Meridian observation of the sun for apparent noon.—With the telescope in the meridian elevated to the sun's altitude, the watch times of transit of the sun's west and east limbs are noted, the mean of which is the watch time of apparent noon; if the observation fails for either limb the reduction to the sun's center is accomplished by adding or subtracting 68 seconds; a refinement in the amount of this time is had by referring to the Ephemeris for the time of the sun's semidiameter passing the meridian for the date of observation; the setting for the approximate altitude of the sun's center is:

$$V \neq 90° - \phi \pm \delta$$

LATITUDE.

Meridian altitude observation of the sun for latitude.—Reverse the sign of "δ" for south declinations:

$$\phi = 90 + \delta - h$$

This observation is conveniently combined with the meridian observation of the sun for time, observing simultaneously the sun's lower and west limbs, recording the watch time and the vertical angle and reversing in the interval of about 2½ minutes, and observing simultaneously the sun's upper and east limbs; the settings for the approximate altitudes of the sun's lower and upper limbs, respectively, are:

$$V \neq 90° - \phi \pm \delta \mp 16'$$

Altitude observation of Polaris at upper culmination for latitude.—

$$\phi = h + \delta - 90°$$

Altitude observation of Polaris at lower culmination for latitude.—The mean time hour angle of Polaris at lower culmination is 11 hours 58 minutes 2 seconds:

$$\phi = h + 90° - \delta$$

The settings for the approximate altitude of Polaris at upper and lower culminations, respectively, are:

$$v \neq \phi \pm (90° - \delta)$$

AZIMUTH.

Altitude observation of the sun for azimuth.—Reverse the signs of "δ" for south declinations:

$$\text{Tan } \tfrac{1}{2} A = \sqrt{\frac{\cos \frac{1}{2}(\zeta+\phi+\delta)\sin \frac{1}{2}(\zeta+\phi-\delta)}{\cos \frac{1}{2}(\zeta-\phi-\delta)\sin \frac{1}{2}(\zeta-\phi+\delta)}}$$

The spherical angles "ζ," "ϕ," and "δ" appear in this equation combined as in the formula for the reduction of an altitude observation of the sun for apparent time, and when it is desired to reduce for both time and azimuth, the above equation for azimuth is to be preferred to any that follow.

Altitude observation of the sun for azimuth.—For south declinations the function "sin δ" becomes negative by virtue of the sine of a negative angle being treated as negative in analytical reductions: If the algebraic sign of the result is positive the azimuth "A" is referred to the north point, but if negative, the azimuth "A" is referred to the south point:

$$\text{Cos } A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

The above equation is very convenient in reducing for azimuth only.

Altitude observation of the sun for azimuth.—To many surveyors the following equation is familiarly expressed directly in terms of the spherical triangle "pole-zenith-sun"; reverse the sign of "δ" for south declinations:

Pole to zenith=90°−ϕ=colat.;
Pole to sun =90°−δ=codecl.;
Zenith to sun=90°−h=coalt.;
S=½ sum of the three sides:

$$\text{Cos } \tfrac{1}{2} A = \sqrt{\frac{\sin S \sin (S - \text{codecl.})}{\sin \text{colat.} \sin \text{coalt.}}}$$

Equal altitude observations of the sun for meridian.—The sun's center at equal altitudes occupies symmetrical positions in azimuth east and west of the meridian in the morning and in the afternoon except for the correction necessary to be applied due to the change in the sun's declination in the interval between the a. m. and p. m. observations:

"$d\,A_\delta$": Correction in azimuth in minutes of angular measure to be applied to the mean position in azimuth to obtain the true south point; the correction is to be applied to the east with a northerly hourly change in declination, or to the west with a southerly hourly change.

"$d\,\delta$": Change in declination of the sun from the a. m. to the p. m. observation, expressed in minutes of angular measure.

"(t_1+t_2)": The sum of the hour angles from apparent noon, or the total watch time from the a. m. to the p. m. observation, expressed in angular measure.

$$d\,A_\delta = \frac{\frac{1}{2}\,d\,\delta}{\cos\,\phi\,\sin\frac{1}{2}\,(t_1+t_2)}$$

The symmetry of the equal altitude observation is preserved by observing opposite limbs in azimuth in the a. m. and p. m. observations, in connection with the same limb in vertical angle in both observations.

With "$\frac{1}{2}\,d\,\delta$" and "$\frac{1}{2}\,(t_1+t_2)$" calculated, the computation can be concluded by applying to "$\frac{1}{2}\,d\,\delta$" the declination coefficient obtained by entering the table of coefficients for computing errors in azimuth due to small errors in declination, arguments: "ϕ" and "$\frac{1}{2}\,(t_1+t_2)$."

Azimuth of Polaris at elongation.—

$$\text{Sin A} = \frac{\cos\,\delta}{\cos\,\phi}$$

A table of azimuths of Polaris at elongation, for latitudes from 25° to 70° N., appears in the Ephemeris, arguments: "δ" and "ϕ."

Azimuth of Polaris at any hour angle.—"t"=sidereal hour angle in angular measure; in hour angles exceeding 90° the function "$-\sin\,\phi\,\cos\,t$" becomes positive by virtue of the cosine of an angle between 90° and 270° being treated as negative in analytical reductions:

$$\text{Tan A} = \frac{\sin\,t}{\cos\,\phi\,\tan\,\delta - \sin\,\phi\,\cos\,t}$$

A table of azimuths of Polaris at all hour angles, for latitudes from 30° to 50° N., appears in the Ephemeris, arguments: "δ," mean time hour angle, and "ϕ."

For other than the above latitudes the surveyor will be required to perform the above analytical solution, accomplished by the following process: convert the usual mean time hour angle into sidereal hour angle, and convert the sidereal hour angle into angular measure to obtain "t" for the above equation.

Polaris at sunset or sunrise.—Polaris is conveniently observed for azimuth by the hour angle method at sunset or sunrise without artificial illumination; the preparation for the observation consists in computing in advance the approximate settings in azimuth and altitude in order to find Polaris, and the plan contemplates an approximate reference meridian: with the time of sunset or sunrise assumed as the time of observation, the hour angle "t" and azimuth "A" are determined in order to find the position of Polaris in azimuth; the position in altitude or vertical angle is found by taking the latitude of the station to which there is to be added or subtracted the angular amount that Polaris is above or below the pole at the assumed hour angle, above for less than 6 hours, and below when over 6 hours; these values are tabulated in the Ephemeris as an adjustment to the elevation of the pole in the table "To find the latitude by an altitude observation of Polaris at any hour angle."

CONVERGENCY OF MERIDIANS.

"m_λ": Measurement along the parallel.
"m_ϕ": Measurement along the meridian.
"a": Equatorial radius of the earth=3963.3 miles=80×3963.3 chains.
"e": Factor of eccentricity, log e=8.915 2515.
"dm_λ": Linear amount of the convergency on the parallel, of two meridians distance apart "m_λ," and distance "m_ϕ" along the meridian: "dm_λ," "m_λ," "m_ϕ" and "a" to be expressed in the same unit:

$$dm_\lambda = \frac{m_\lambda\,m_\phi}{a}\,\tan\,\phi\,\sqrt{1-e^2\sin^2\phi}$$

TABLE 26.—INCREMENTS OF CURVATURE

Change in azimuth due to convergency of meridians

Lat.	Departure in chains										Lat.
	10	20	30	40	50	60	70	80	90	100	
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	°
25	0 3	0 6	0 9	0 12	0 15	0 18	0 21	0 24	0 27	0 30	25
26	0 3	0 6	0 10	0 13	0 16	0 19	0 22	0 25	0 28	0 32	26
27	0 3	0 7	0 10	0 13	0 17	0 20	0 23	0 26	0 30	0 33	27
28	0 4	0 7	0 10	0 14	0 17	0 21	0 24	0 28	0 31	0 35	28
29	0 4	0 7	0 11	0 14	0 18	0 22	0 25	0 29	0 32	0 36	29
30	0 4	0 8	0 11	0 15	0 19	0 22	0 26	0 30	0 34	0 38	30
31	0 4	0 8	0 12	0 16	0 20	0 23	0 27	0 31	0 35	0 39	31
32	0 4	0 8	0 12	0 16	0 20	0 24	0 28	0 32	0 37	0 41	32
33	0 4	0 8	0 13	0 17	0 21	0 25	0 30	0 34	0 38	0 42	33
34	0 4	0 9	0 13	0 18	0 22	0 26	0 31	0 35	0 39	0 44	34
35	0 5	0 9	0 14	0 18	0 23	0 27	0 32	0 36	0 41	0 46	35
36	0 5	0 9	0 14	0 19	0 24	0 28	0 33	0 38	0 42	0 47	36
37	0 5	0 10	0 15	0 20	0 24	0 29	0 34	0 39	0 44	0 49	37
38	0 5	0 10	0 15	0 20	0 25	0 30	0 36	0 41	0 46	0 51	38
39	0 5	0 10	0 16	0 21	0 26	0 32	0 37	0 42	0 47	0 53	39
40	0 6	0 11	0 16	0 22	0 27	0 33	0 38	0 44	0 49	0 54	40
41	0 6	0 11	0 17	0 23	0 28	0 34	0 40	0 45	0 51	0 56	41
42	0 6	0 12	0 18	0 23	0 29	0 35	0 41	0 47	0 53	0 58	42
43	0 6	0 12	0 18	0 24	0 30	0 36	0 42	0 48	0 54	1 1	43
44	0 6	0 12	0 19	0 25	0 31	0 38	0 44	0 50	0 56	1 3	44
45	0 6	0 13	0 20	0 26	0 32	0 39	0 46	0 52	0 58	1 5	45
46	0 7	0 13	0 20	0 27	0 34	0 40	0 47	0 54	1 0	1 7	46
47	0 7	0 14	0 21	0 28	0 35	0 42	0 49	0 56	1 3	1 10	47
48	0 7	0 14	0 22	0 29	0 36	0 43	0 50	0 58	1 5	1 12	48
49	0 8	0 15	0 22	0 30	0 37	0 45	0 52	1 0	1 7	1 15	49
50	0 8	0 16	0 23	0 30	0 39	0 46	0 54	1 2	1 10	1 17	50
51	0 8	0 16	0 24	0 32	0 40	0 48	0 56	1 4	1 12	1 20	51
52	0 8	0 17	0 25	0 33	0 42	0 50	0 58	1 6	1 15	1 23	52
53	0 9	0 17	0 26	0 34	0 43	0 52	1 0	1 9	1 18	1 26	53
54	0 9	0 18	0 27	0 36	0 45	0 54	1 2	1 12	1 20	1 29	54
55	0 9	0 18	0 28	0 37	0 46	0 56	1 5	1 14	1 23	1 33	55
56	0 10	0 19	0 29	0 38	0 48	0 58	1 7	1 17	1 27	1 36	56
57	0 10	0 20	0 30	0 40	0 50	1 0	1 10	1 20	1 30	1 40	57
58	0 10	0 21	0 31	0 42	0 52	1 2	1 13	1 23	1 34	1 44	58
59	0 11	0 22	0 32	0 43	0 54	1 5	1 16	1 26	1 37	1 48	59
60	0 11	0 22	0 34	0 45	0 56	1 7	1 19	1 30	1 41	1 52	60
61	0 12	0 23	0 35	0 47	0 58	1 10	1 22	1 34	1 45	1 57	61
62	0 12	0 24	0 37	0 49	1 1	1 13	1 25	1 38	1 50	2 2	62
63	0 13	0 26	0 38	0 51	1 4	1 16	1 29	1 42	1 55	2 7	63
64	0 13	0 27	0 40	0 53	1 6	1 20	1 33	1 46	2 0	2 13	64
65	0 14	0 28	0 42	0 56	1 10	1 24	1 37	1 51	2 5	2 19	65
66	0 15	0 29	0 44	0 58	1 13	1 27	1 42	1 57	2 11	2 26	66
67	0 15	0 31	0 46	1 1	1 16	1 32	1 47	2 2	2 18	2 33	67
68	0 16	0 32	0 48	1 4	1 20	1 36	1 52	2 8	2 24	2 40	68
69	0 17	0 34	0 51	1 8	1 24	1 41	1 58	2 15	2 32	2 49	69
70	0 18	0 36	0 54	1 11	1 29	1 47	2 5	2 23	2 40	2 58	70

TABLE 27.—THE ARPENT AND VARA UNITS.[1]

ARPENTS TO ACRES

The arpent (*arpen*) is an old French land measure of *area* whose use was employed wherever there were French settlements in North America. The arpent seems never to have been used directly as a linear measure, but tracts of land were frequently described in length and breadth in terms of arpents, the unit intended being the length of the side of a square arpent. The values given below were employed with considerable uniformity, differing only slightly as to exactness, but with the distinction as noted.

The value in Louisiana, Mississippi, Alabama, and Northwestern Florida:

1 arpent=0.84625 acres (very nearly).
The side of a square arpent=2.909 chains=191.994 feet.

The value in Arkansas and Missouri:

1 arpent=0.8507 acres.
The side of a square arpent=2.91667 chains=192.500 feet.

VARAS TO CHAINS AND FEET

The vara is a Spanish and Mexican unit of linear measurement.

The value in the public domain of the Southwest:

1 vara=32.99312 inches=4.1658 links.
100 varas=4.1658 chains=274.943 feet.

The value in Florida:

1 vara=33.372 inches=4.2136 links.
100 varas=4.2136 chains=278.100 feet.

The value in Texas:

1 vara=33.333333 inches=4.2088 links.
100 varas=4.208754 chains=277,777 feet.
36 varas=1.5152 chains=100.000 feet.
1900.8 varas=80.00 chains=5280 feet=1 mile.
75.13 varas square=5645.375 square varas=1 acre.

[1] In some cases slightly different values were employed in the boundary surveys of the French, Spanish, and Mexican land grants in the several surveying districts, due no doubt to the lack of exact standards, the disposition to continue the established local practices, and more or less on account of the use of approximate conversion factors, so that if it is necessary to ascertain the authority for definite equivalents an examination should be made of the early surveying records and court opinions of that particular district.

TABLE 28.—METRIC CONVERSIONS

CONVERSION TABLES

Metric Linear Units

ONE METER is equivalent to:

Links	Feet	Inches
4.970 960	3.280 833	39.37
Log. 0.696 440	Log. 0.515 984	Log. 1.595 165

LINKS TO FEET TO METERS

Links	Feet	Meters
1	.660	.201 168
2	1.320	.402 337
3	1.980	.603 505
4	2.640	.804 674
5	3.300	1.005 842
6	3.960	1.207 010
7	4.620	1.408 179
8	5.280	1.609 347
9	5.940	1.810 516
10	6.600	2.011 684

METERS TO FEET TO LINKS

Meters	Feet	Links
1	3.280 833	4.970 960
2	6.561 667	9.941 919
3	9.842 500	14.912 879
4	13.123 333	19.883 838
5	16.404 167	24.854 798
6	19.685 000	29.825 758
7	22.965 833	34.796 717
8	26.246 667	39.767 677
9	29.527 500	44.738 636
10	32.808 333	49.709 596

FEET TO LINKS TO METERS

Feet	Links	Meters
1	1.515 152	.304 801
2	3.030 303	.609 601
3	4.545 454	.914 402
4	6.060 606	1.219 202
5	7.575 758	1.524 003
6	9.090 909	1.828 804
7	10.606 060	2.133 604
8	12.121 212	2.438 405
9	13.636 364	2.743 205
10	15.151 515	3.048 006